

# A brief analysis of the physical significance of Heisenberg's uncertainty principle

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**Abstract.** The principle of uncertainty is one of the essential principles of quantum mechanics as it outlines that there is a reflective limit to the measurement of microscopic particle motion. However it is impossible to measure at once and accurately both the position and momentum data of any microscopic particle no matter what means are used to do so. This paper starts by discussing the notion and mathematical derivation of the uncertainty principle as put forth by Heisenberg explaining the physical relevance of the principle. It introduces the natural uncertainty the principle introduces to the dynamics of particles in the microscopic world, and the significant part it plays in the theory of quantum mechanics. The article overviews the explanations of the principle on phenomena including atomic stability and scanning tunneling microscopes, and the interference of measurement actions on the states of particles and the shortcomings of measurement precision. According to the wave-particle duality of quantum mechanics, this article gives a required theoretical basis of microscopic phenomena. Likewise, the use of the research theory on the uncertainty principle by distinguished scholars in the physics fraternity helps the readers in achieving the knowledge in related subjects more easily.

**Keywords:** uncertainty, Heisenberg limit, wave-particle duality, measurement

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## 1. Introduction

As the traditional classical mechanics evolved, individuals in the past had a tendency to assume that the world was uniquely fixed, and that all the unfamiliar problems could be blamed on the incompleteness and inadequacy of scientific development. As a result of the development of the modern sciences, the concept of matter shifted to the microscopic, and individuals succeeded in knowing that the elusive uncertainty is the more common feature in the objective world [1]. As a reaction against the evasiveness of the microscopic world, Werner Karl Heisenberg, in 1927, came up with the uncertainty principle. According to this recommendation, the determinism of the macroscopic world of classical mechanics is not the case with the motion of the microscopic particles; rather, it is uncertain in the sense of probabilities. By examining the motion of particles Heisenberg in a quantitative manner indicated the measurement limits required to measure the motion of particles. In addition, at the point when the gist of uncertainty in the microscopic world was deciphered, this theory that turned a departure of conventional ideas made many scholars eager to further explore, thus facilitating the creation and development of the quantum mechanics. Hawking praised

uncertainty principle by Heisenberg as a necessity, inevitable feature of the world and it has far reaching implications on the perceptions that people have of the world [1].

This paper will start with the uncertainty principle Heisenberg to continue discussing the realized nature and physical importance of uncertainty in the microscopic world among people who want to comprehend Heisenberg uncertainty principle, this article will help them to have a better understanding of the uncertainty of the microscopic world.

## 2. The fundamental concept of the Heisenberg uncertainty principle

### 2.1. The historical development and context of Heisenberg's uncertainty principle

From 1925 to 1927, Heisenberg was contemplating the interpretational analysis within matrix mechanics. He realized that in the microscopic world, the measurement process of particles inevitably affects their state, leading to the impossibility of simultaneously determining the position and momentum of particles with precision. In response, Heisenberg in 1927 provided the original heuristic formulation of the uncertainty principle. The principle states that in a quantum mechanical system, the position and momentum of a moving particle cannot be determined simultaneously. The more precisely the position is measured, the less precisely the momentum can be measured, and vice versa. Moreover, the product of the uncertainty in position and the uncertainty in momentum is not less than a fixed value. In the same year, Kennard (Earle Hesse Kennard) clarified the relationship between the product of the standard deviations of position and momentum and the reduced Planck constant, proving that  $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ . Among them,  $\sigma_x$  and  $\sigma_p$  represent the standard deviations of particle position and momentum, respectively. Kennard's discovery provided a more precise mathematical formulation of the uncertainty principle [2]. In the same year, Niels Bohr proposed the "complementarity principle." The complementarity principle states that wave-particle duality is manifested through the mutual supplementation of wave-like and particle-like properties, which are complementary images in the process of motion. Bohr further revealed the essence of uncertainty in the microscopic world regarding particle measurement. From the perspective of microscopic quantum mechanics, any physical quantity or property does not inherently exist but only becomes meaningful when the object is measured. At this point, the uncertainty principle also reveals that the behavior of microscopic objects depends on the nature of observational conditions [3].

### 2.2. The concept and mathematical derivation of Heisenberg's uncertainty principle

The Heisenberg Uncertainty Principle states that the position and momentum of particles in the microscopic world cannot be measured simultaneously. Its expression is  $\Delta x \Delta p \geq \frac{\hbar}{2}$ ,  $\Delta x$  represents the uncertainty of the particle position,  $\Delta p$  represents the uncertainty in a particle's momentum ( $\hbar = 1.054 \times 10^{-34}$  J.s). Different from the average in classical mechanics, the average in quantum mechanics typically refers to the expectation value of a mechanical quantity on a quantum state, which is obtained by applying the operator corresponding to the physical quantity to the superposition state and then taking the inner product with the quantum state [4].

Assume a wave function  $\psi(x, t)$  describing the state of a quantum system, and the positional operator  $\hat{x} = x$  expected value expression of the position can be obtained by acting on the wave function:

$\langle x \rangle = \int \psi^*(x, t) \hat{x} \psi(x, t) dx$  at the same time, the momentum operator  $\hat{p} = -i\hbar \frac{\partial}{\partial x}$ , when applied to the wave function, can obtain the expression for the expected value of momentum  $\langle p \rangle = \int \psi^*(x, t) \hat{p} \psi(x, t) dx$ .

In order to study the properties of quantum systems from a statistical point of view, the uncertainty is usually

expressed in the form of mean square deviation. That is, positional uncertainty  $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$  and the uncertainty in momentum  $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$ . Similarly, two new equations can be derived.  $\langle x^2 \rangle = \int \psi^*(x, t) \widehat{x^2} \psi(x, t) dx$  and  $\langle p^2 \rangle = \int \psi^*(x, t) \widehat{p^2} \psi(x, t) dx$ .

This article chooses a more defined method for derivation, while considering that Poisson distribution is a coherent state that reflects the minimum uncertainty, while Gaussian distribution can be regarded as the Poisson distribution in the limit case. Therefore, the author will use the definition method to derive the minimum value of uncertainty relationship using Gaussian wave packet form wave function as an example.

Consider Gaussian wave packet form wave function

$$\psi(x, t) = A e^{-\frac{(x-x_0)^2}{4\sigma_x^2} + i\frac{p_0}{\hbar}x - i\omega t} \quad (1)$$

Among them,  $A$  is the normalization constant,  $x_0$  representing the position center,  $p_0$  is the momentum center, representing the momentum of the particle,  $\omega$  is the angular frequency. Put  $\widehat{x}$  and  $\widehat{x^2}$  into the formula respectively.

Calculate this result:

$$\langle x \rangle = x_0, \langle x^2 \rangle = x_0^2 + |A|^2 \sqrt{\pi} \sigma_x^3 \quad (2)$$

Calculated through normalization

$$A = \frac{1}{\sqrt{\sigma_x \sqrt{\pi}}} \quad (3)$$

The positional uncertainty is obtained as

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{x_0^2 + \left(\frac{1}{\sqrt{\sigma_x \sqrt{\pi}}}\right)^2 \sqrt{\pi} \sigma_x^3 - x_0^2} = \sigma_x \quad (4)$$

In a similar way, put  $\widehat{p}$  and  $\widehat{p^2}$  into the formula respectively. Calculate this result:

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{p_0^2 + \frac{\hbar^2}{4\sigma_x^2} - p_0^2} = \sqrt{\frac{\hbar^2}{4\sigma_x^2}} \quad (5)$$

Multiplying the results from both of them

$$\Delta x \Delta p = \sigma_x \sqrt{\frac{\hbar^2}{4\sigma_x^2}} = \frac{\hbar}{2} \quad (6)$$

This is the minimum, the measurement of microscopic particles [4].

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (7)$$

### 3. Heisenberg uncertainty principle: physical significance as theory

#### 3.1. The importance of discovering the motion of particles in the microscopic world

##### 3.1.1. Fundamental properties of microscopic particles

Within the macroscopic world, people used to assume that by using sufficiently fine instruments, one would measure at the same time the position and momentum of an object. But in the microscopic level there is no

such determinism, particles have wave-particle duality, and cannot be explained by classical mechanics. The Heisenberg uncertainty principle argues that a particle can never be considered in one place and at one time possessing a definite momentum. This indeterminacy is not explained by the impossibility to use more sophisticated observational techniques, but rather by the inherent characteristics of particles. This uncertainty can be neglected at the macroscopic scale since the value of Planck constant is extremely small but at the microscopic scale, exercise to elucidate the behavior of particles has to involve the probability wave functions, hence encouraging the creation of quantum mechanics [5].

### *3.1.2. Wave-particle duality in quantum mechanics*

The uncertainty principle is proved through experiments which measure the diffraction of electrons, and neutral potassium atoms in any experiment and concludes that there is a wave-particle duality in microscopic particles. In single-slit diffraction, in passing through a thin slit, microscopic particles exhibit diffraction fringes on a screen resembling waves on a screen, which show the wave-like behavior of particles [1]. Once the microscopic particles are passed through an even narrower slit, the uncertainty of their position diminishes, but the uncertainty of their momentum, and the diffraction pattern is widened. In electron, double-slit interference example, trying to know which slit the electron is going through kills its wave-like characteristics causing the interference fringes to disappear [1]. This proves that the characteristics of a particle and a wave are mutually exclusive and this goes further to confirm the uncertainty principle given by Heisenberg which states that a particle cannot be measured in its exact position and at the same time in its exact momentum.

## 3.2. Implications in the particle measurement in the microscopic world

### *3.2.1. Mutual interference in measurements*

The principle of mutual interference in measurements is that a measurement is sensitive to the version of the product under test when it is taken.

In the measurement of microscopic particles, the very process of measurement causes a perturbation of the state of the particle. The better a particle position is measured, the more the change in its quantum state is important [6]. As an example, in observing an electron with light, the maximum accuracy that can be attained in the measurement is dependent on the wavelength of the light. The change in the momentum of the electron at the moment position is measured, i.e. when the photon is scattered off by the electron, is discontinuous [1]. A short wavelength light provides a more accurate position of the electron to measure, but at this stage, the photon is of a greater energy and once it collided with the electron it gave a greater error in measuring the electrons momentum. On the same note, with the light that has a longer wavelength, one can more accurately figure out the state of momentum of the electron at a certain time but at this point, however, the longer wavelength light is going to cause an error in the determination of the position of the electron. It means that it is impossible to measure the position and momentum of a particle simultaneously with high accuracy and the action of microscopic particles does not correspond to the classical concepts of physics. Even in the measurement of small reservoirs, the electromagnetic and gravitational fields of measuring apparatus also interfere with the measurements.

### *3.2.2. Accuracy limits of measurement*

The uncertainty relation provides the maximum that the measuring accuracy can have and it is impossible to surpass the maximum. This assumes a significant instructive role in the comprehending of the results of microscopic physical tests, and the precision of measurements, in the phase of performing microscopic tests, the Heisenberg uncertainty rule can be applied to obtain the suitable order of magnitude of the error, so as to evaluate the integrity of the data. Quantum precision measurement is also one critical area of the research in current quantum information technology. The multiple measurements are usually used to determine the

precision. The higher the number of quantum states, the lower will be the error of estimation, this is known as the standard quantum limit, and the Heisenberg uncertainty relation presents an extreme value of the maximum accuracy possible by a measurement, which will also require non-classical correlations like quantum entanglement, thus, with the help of the Heisenberg uncertainty principle, quantum resources can be exploited to overcome the confinement of the classical limit and propel the accuracy of a measurement to go up [7].

## **4. The Heisenberg uncertainty principle finds use in the following applications**

### **4.1. Illustration of atomic stability**

Classical mechanics explain that a spinning by the nucleus of with an orbital nature, continuously emits electromagnetic waves and ultimately crashes into the nucleus because of loss of energy. But this theory fails to be equivalent to the fact of the prevalence of stationary atoms. This can be totally explained by the uncertainty principle of Heisenberg. The particles are spread around the nucleus in the probability waves. The possibility of an electron to be localized with respect to the nucleus leads to the smaller the uncertainty in position, and the greater the uncertainty in momentum, so the minimum energy there can be is very large. This implies that it is only in case of the action of vast quantity of energy on the outer electron that the electron may be in the condition to fall into the nucleus. In normal conditions, there is no such enormous energy to cause the electron to fall into the nucleus so that such a situation does not take place and makes the atom stable.

### **4.2. Scanning Tunneling Microscope and the process midway quantum tunneling effect**

A Scanning Tunneling Microscope (STM) is a device dependent on the effect of quantum tunneling. Its action is dependent on the voltage between a sharp metal probe and the surface of a sample. Through the processes of quantum tunneling the electrons may tunnel between the probe and the sample when the probe is brought one proximal to the sample surface producing a tunneling current. The topography of the sample on the surface can be determined by quantifying the range of such current and therefore, high-resolution scanning is possible. The problem with classical mechanics is that it does not explain how particles with low energy can cross a potential barrier, but according to quantum mechanics, the wave function calculates the state of the particles and position and momentum near the potential barrier are not known. The energy of the particle can be higher than the barrier over short periods, which then leads to quantum tunneling.

### **4.3. The future and the prospects of high-precision measurement technology**

As modern physics has grown further, experimental physicists have also investigated the microscopic scale and accepted the Heisenberg limit in the real accuracy of microscopic measurements of particles, providing course towards the high-level of measuring technology. Not only in mechanical and optical systems, the high-precision measurement technology has chances of being used, but in weak force detection and dark matter searches. As an example, measurement schemes are built on the large photon-number Fock states: photon Fock states are employed to characterize a quantum state of a light field or a quantized electron field: with orthogonality and completeness. Quantum metrology involves the use of non-classical states, and typically the Fock states with definite photon numbers to gain benefits over classical traditional methods [8]. To enhance the sensitivity of the detector, people frequently operate large number of photon Fock state since the detection sensitivity critically depends on the small displacement or changes in phase shift between electron fields in a microwave. Programmable photon number filters, including those in Teams of Hefei National Laboratory and

Shenzhen International Quantum Research Institute have been used to produce Fock states with up to 100 photons in superconducting microwave cavities, showing quantum-enhanced metrology with 40-photon Fock state near to the Heisenberg scale to obtain up to a maximum 14.8 dB of metrology. Not only does the proposal of the Heisenberg limit contribute to the essence of the microscopic particles, but new directions and ideas on the development of quantum precision measurement technology are also given.

## 5. Conclusion

The Heisenberg uncertainty principle is a fundamental theory of quantum mechanics that can also be interpreted so that it makes clearer that there is uncertainty in the microscopic world. This paper examines the history and background of the uncertainty principle, considers its rich connotations, and points out the weaknesses of the classical mechanics at the microscopic scale. As to the motion of microscopic particles, the Heisenberg uncertainty principle not only claims that uncertainty is a property of microscopic particles, but also gives an account of how the behavior of measurements affects the microscopic world, and constitutes an ultimate limit to the low-level accuracy of measurements in principle.

This theory not only disputes the classical ideas of classical physics but also creates the quantum mechanics towards the future. It gives theoretical explanations to tunneling reactions in microscopic facts that are unaccounted by classical physics apparatus like atomic stability. Besides, it has led to the development of technologies such as the scanning electron microscope and the development of the semiconductor sector. The Heisenberg uncertainty principle promises wide opportunities. As high-precision measuring technology advances and more in-depth research is conducted in the microscopic quantum world, they will be more useful resources to humanity in understanding the natural world.

## References

- [1] Lu, Y. (2019). On the uncertainty principle and uncertainty. *Science and Technology Innovation and Application*, (16), 53–54.
- [2] Cui, H. (2019). On the rationality and validity of the Kennard inequality. *American Journal of Modern Physics*, (16), 53–54.
- [3] Li, Q. (2010). On Bohr's complementarity principle. *Journal of Bijie University*, 28(4), 83–86.
- [4] Yuan, A., & Liu, D. (2011). A brief analysis of the uncertainty principle in quantum mechanics. *College Physics*, 30(11), 44–49.
- [5] Wu, Y. (2018). Interpretation and application of the uncertainty principle. *China High-Tech*, (18), 112–114.
- [6] Zhang, Y. (2016). *Quantum measurement and its back-action* [Doctoral dissertation, University of Science and Technology of China].
- [7] Wang, B., & Zhang, L. (2024). Progress in photonics quantum metrology. *Chinese Journal of Lasers*, 51(1), Article 0112001.
- [8] Deng, X., Li, S., Chen, Z.-J., Ni, Z., Cai, Y., Mai, J., Zhang, L., Zheng, P., Yu, H., Zou, C.-L., Liu, S., Yan, F., Xu, Y., & Yu, D. (2024). Quantum-enhanced metrology with large Fock states. *Nature Physics*, 20(12), 1874–1880.