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Hierarchical hybrid Ising simulation solution method for multi-agent task allocation and collaborative path planning

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Abstract. Addressing the challenges inherent in complex intelligent planning—specifically high-dimensional discrete decision spaces, strong constraint coupling, and the propensity of traditional single-layer Ising/Quadratic Unconstrained Binary Optimization (QUBO) modeling to induce variable inflation and penalty imbalance—this paper proposes a hierarchical hybrid Ising simulation framework. This framework is investigated through its application to the problem of multi-agent task assignment and collaborative path planning. The proposed method models task assignment and visit sequencing as an outer-layer QUBO/Ising optimization problem, incorporating techniques such as continuous relaxation, momentum enhancement, noise perturbation, and feedback correction to bolster search capabilities. Concurrently, the inner layer handles path construction, conflict detection, and local repair, feeding the repair information back to the outer layer to establish a closed-loop optimization process. Experimental results demonstrate that this method outperforms comparative approaches across key metrics, including total cost, feasible solution rate, conflict control, and convergence stability.

Keywords: complex intelligent planning, QUBO, hierarchical hybrid solving, hybrid annealing, multi-agent coordination, path planning

1. Introduction

Complex intelligent planning problems are widely prevalent in scenarios such as multi-robot collaboration, unmanned system scheduling, intelligent logistics and distribution, and traffic route optimization [1-3]. Typically, it simultaneously involves discrete decision-making, spatiotemporal coupling, and multi-constraint coordination, exhibiting the characteristic features of high-dimensional combinatorial optimization. As the scale of tasks, environmental complexity, and the number of agents increase, traditional exact search methods or single-heuristic approaches often face challenges such as high computational overhead, susceptibility to local optima, and insufficient scalability, making it difficult to obtain high-quality, feasible solutions [4].

In recent years, the Ising model and its corresponding QUBO modeling approach have garnered attention due to their strong expressive power for combinatorial optimization [5, 6]. Discrete decisions can be mapped to spin variables and solved by combining annealing or continuous dynamics mechanisms [7-9]. However, existing research largely focuses on standard combinatorial optimization problems, paying insufficient attention to constraints—such as task sequencing, path conflicts, time windows, and collaborative collision avoidance—within the context of complex intelligent planning [10-12]. If the aforementioned factors are encoded simultaneously into a single-layer QUBO model, it typically leads to variable bloat, difficulties in balancing penalty terms, and an excessively dense coupling matrix—consequently compromising solution efficiency [13].

To address the aforementioned issues, this paper proposes a hierarchical hybrid Ising simulation framework designed for complex intelligent planning. This approach models task allocation and coarse-grained access sequencing as an outer-layer QUBO/Ising optimization problem, constructing an enhanced hybrid annealing mechanism through continuous relaxation, momentum memory, noise injection, and feedback correction. Simultaneously, path construction, conflict detection, and local repair are situated within an inner-layer module, with evaluation results fed back to the outer layer to form a closed-loop optimization process characterized by "outer-layer search—inner-layer repair—feedback update."

The main contributions of this paper are as follows: First, we propose a hierarchical hybrid Ising modeling framework that effectively decouples discrete core decision-making from complex execution constraints. Second, we design an enhanced hybrid annealing mechanism to improve the stability and global optimization capabilities of the outer-layer search. Third, we validate the effectiveness of the proposed method through experiments on multi-agent task assignment and cooperative path planning; the results demonstrate that this method outperforms traditional simulated annealing, standard Ising methods, and a feedback-free hierarchical variant in terms of total cost, feasible solution rate, conflict control, and scalability.

2. Basic principles

As illustrated in Figure 1, addressing the challenge in complex intelligent planning problems—where discrete decision-making and intricate execution constraints are tightly coupled and difficult to directly map into a unified, single-layer QUBO/Ising model—this paper proposes a hierarchical hybrid Ising simulation framework. Such problems are prevalent across various scenarios, including multi-robot task allocation, collaborative UAV reconnaissance, intelligent logistics scheduling, and multi-vehicle path coordination; they not only exhibit distinct characteristics of combinatorial optimization but are also accompanied by constraints such as path conflicts, time windows, capacity limits, and multi-agent collaborative relationships. Attempting to unfold all constraints simultaneously into a single-layer Quadratic Unconstrained Binary Optimization (QUBO) model typically results in a rapid explosion in the number of variables, difficulties in balancing penalty terms, and a densification of the coupling matrix—consequently diminishing both modeling and solution efficiency. To mitigate these issues, this paper adopts a hierarchical modeling paradigm characterized by an "outer-layer discrete search—inner-layer path repair—feedback update" workflow: the outer layer handles task allocation and coarse-grained sequence optimization, while the inner layer manages path construction, conflict detection, and local repair, feeding the repair results back to the outer layer to establish a closed-loop optimization process.

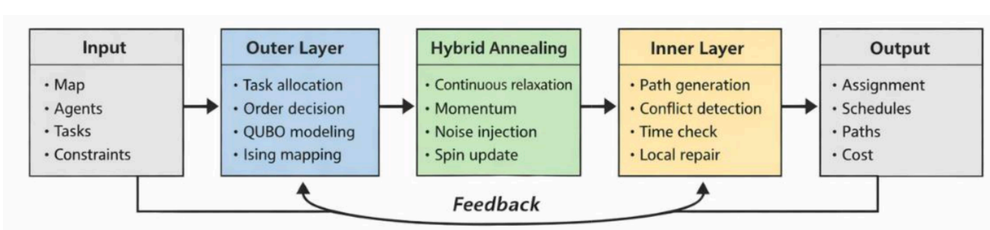

Figure 1. A hierarchical hybrid ising simulation framework for solving complex intelligent planning problems

Consider a discrete environment represented as a directed graph $G = (V, E)$, where V is the set of nodes and E is the set of edges. The edge weight c_{ij} denotes the movement cost from node v_i to node v_j . The set of agents is denoted as $A = \{1,2,\ldots,M\}$, and the set of tasks is denoted as $T = \{1,2,\ldots,K\}$. Each agent $a \in A$ has a start point s_a , a goal point g_a , and a capacity upper bound q_a . Each task $t \in T$ corresponds to a task node $v(t)$. In the outer-layer modeling, only the task assignment variables and the ordering variables are retained, and the objective function is expressed as:

$$E_{\text{QUBO}} = E_{\text{cost}} + P_{\text{task}} + P_{\text{order}} + P_{\text{link}} + P_{\text{balance}} \tag{1}$$

Among them, E_{cost} takes into account both the task assignment cost and the cost of transitioning between consecutive tasks. P_{task} , P_{order} , P_{link} , and P_{balance} are used to enforce the constraints of unique task assignment, order consistency, logical consistency between assignment and ordering, and load balancing, respectively. After expanding the outer-layer binary variables into a vector $u \in \{0,1\}^n$, we have:

$$E_{\text{QUBO}}(u) = u^T Q u + b^T u + c. \tag{2}$$

Further, through variable substitution

$$s_i = 2u_i - 1, s_i \in \{-1,1\}, \tag{3}$$

it can be transformed into a standard Ising Hamiltonian.

$$H\left(s\right) = -\sum_i h_i s_i - \sum_{i<j} J_{ij} s_i s_j. \tag{4}$$

It should be noted that complex constraints such as path conflicts, edge conflicts, and time windows are not directly incorporated into the outer-layer QUBO, but are instead handled by the inner-layer module, thereby avoiding the variable explosion caused by the explicit introduction of path-level variables.

To enhance the outer-layer search capability, this paper introduces momentum memory, noise injection, and feedback correction based on continuous relaxation Ising dynamics, thereby constructing an enhanced hybrid annealing mechanism. Define the continuous state vector $y_t \in [-1,1]^n$, whose update rule is given as follows:

$$y_{t+1} = y_t - \eta_t \nabla \widetilde{H}(y_t) + \beta_t m_t + \sigma_t \xi_t + \rho_t r_t, \tag{5}$$

where η_t is the step size, m_t is the momentum term, ξ_t is the noise term, and r_t is the inner-layer feedback term, with β_t , σ_t , and ρ_t controlling the respective intensities. The gradient term drives the energy descent, the momentum term improves search continuity and stability, the noise term enhances the ability to escape local minima, and the feedback term introduces inner-layer feasibility information into the outer-layer search, thereby extending the optimization objective from pure energy minimization to joint "energy-feasibility" guidance. After each update, the continuous state is projected onto the interval $[-1,1]^n$, and binarization is performed via the sign function to obtain a candidate discrete solution.

For a candidate solution yielded by the outer layer, the inner-layer module first extracts the task sequence of each agent, and constructs a complete execution path using the shortest path method with the respective start and goal points as boundaries. Subsequently, it detects node conflicts, edge conflicts, time window violations, and capacity violations, and employs strategies such as wait insertion, local exchange, task reassignment, and local replanning to perform repair. To quantify the repair cost, define:

$$R(x) = \omega_1 N_{\text{conflict}} + \omega_2 N_{\text{swap}} + \omega_3 T_{\text{delay}}, \tag{6}$$

and further construct a comprehensive evaluation function.

$$\widetilde{E}(x) = E_{\text{outer}}(x) + \lambda_r R(x). \tag{7}$$

Therefore, the quality of a candidate solution depends not only on the outer-layer Ising energy but also on the repair cost required to convert it into an executable plan. Meanwhile, the inner layer can generate feedback terms r_t based on frequent conflict patterns, which are used to suppress the recurrence of similar conflicts in subsequent iterations.

In summary, this paper constructs a hierarchical hybrid solution framework with outer-layer Ising/QUBO search as the core, inner-layer repair and evaluation as a supplement, and the feedback loop as a link. Its basic workflow includes: precomputing shortest-path costs, establishing the outer-layer QUBO/Ising model, initializing the continuous state, performing outer-layer hybrid annealing updates, completing path repair and evaluation in the inner layer, adjusting the outer-layer search based on feedback, and iterating until a feasible planning solution with optimal comprehensive evaluation is obtained. This framework effectively alleviates the variable explosion problem caused by directly incorporating complex planning constraints into the QUBO formulation, while preserving the combinatorial optimization expression power of the Ising model, thereby providing a theoretical foundation for subsequent experimental validation.

3. Experimental results and discussion

To validate the effectiveness of the proposed hierarchical hybrid Ising simulation framework for complex intelligent planning problems, this paper conducts simulation experiments on multi-agent task assignment and cooperative path planning problems in a discrete two-dimensional grid environment. Three maps, namely , , and , are adopted in the experiments, with obstacle densities set to 10%, 20%, and 30%, respectively. The number of agents is taken as , and the number of tasks as . Task points and agent start and goal positions are randomly generated in traversable regions, and both the assignment cost and the task transition cost are precomputed as shortest path lengths. For different combinations of problem scale and complexity, multiple random instances are independently generated. Under the same computational budget, the proposed method is compared with traditional Simulated Annealing (SA), Greedy Local Search (GLS), Plain Ising Solving (Plain Ising), and a hierarchical version without feedback. Evaluation metrics include total cost, feasible solution rate, average number of conflicts, average number of iterations to convergence, and average runtime. The multi-agent cooperative path planning results obtained by different methods in a typical scenario are shown in Figure 2, and the convergence processes of different methods in a typical scenario are shown in Figure 3.

As can be seen from Figure 2 and 3, the experimental results indicate that the proposed method achieves better or more robust comprehensive performance in most scenarios, and its advantages become more pronounced as the number of tasks increases, obstacle density rises, and multi-agent conflicts become more frequent. Compared with SA and GLS, the proposed method achieves a better balance among task assignment, path transition, and temporal coordination, thereby obtaining a lower total cost. Compared with Plain Ising, its advantages are mainly reflected in a higher feasible solution rate and more stable low-cost output in complex scenarios, indicating that although a single-layer Ising energy descent possesses certain search capability, it cannot adequately capture the execution feasibility in complex planning. The hierarchical repair and feedback mechanism effectively suppresses undesirable solution patterns characterized by low outer-layer energy but high actual execution cost. Although the proposed method incurs a slight increase in per-iteration overhead due to the introduction of inner-layer repair, the overall increase in runtime remains within an acceptable range. Considering its ability to reduce ineffective searches and significantly improve the feasible solution rate, its overall efficiency in obtaining executable high-quality solutions is still superior to that of traditional methods.

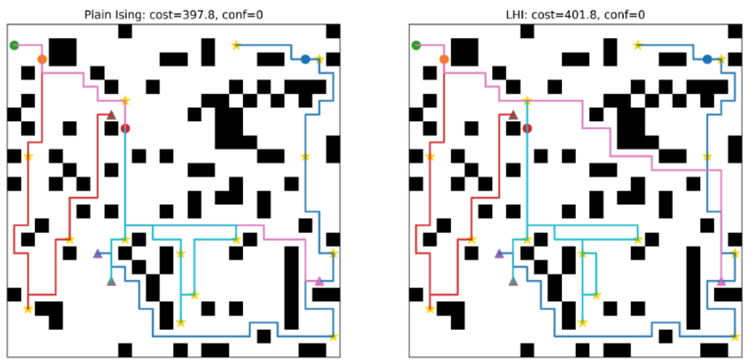

Figure 2. Comparison of multi-agent cooperative path planning results in a typical scenario

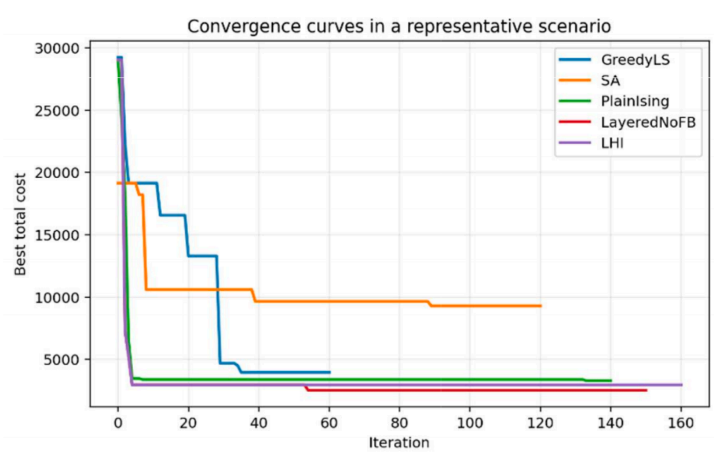

Figure 3. Comparison of convergence curves of different solution methods in a typical scenario

As can be seen from Figure 4 and 5, further ablation experiments show that the momentum term primarily improves search continuity and convergence stability, the noise term enhances the ability to escape local minima, the hierarchical repair mechanism significantly increases the feasibility of solutions, and the feedback mechanism further converts inner-layer feasibility information into guidance for the outer-layer search. Among these, when relying solely on outer-layer enhanced annealing without introducing hierarchical repair, although the outer-layer QUBO energy of some instances is low, the final feasible solution rate and comprehensive total cost are unsatisfactory, indicating that the hierarchical structure is key for the proposed method to accommodate complex planning constraints. The scaling experiments also show that as the number of tasks, the number of agents, or the obstacle density increases, the total cost and runtime of all methods increase, but the proposed method exhibits a smoother performance degradation trend. This indicates that the design of preserving discrete combinatorial structures in the outer layer while absorbing path-level and temporal constraints in the inner layer can, to a certain extent, avoid the rapid explosion of variable and coupling scales in single-layer modeling, thereby maintaining good scalability and robustness.

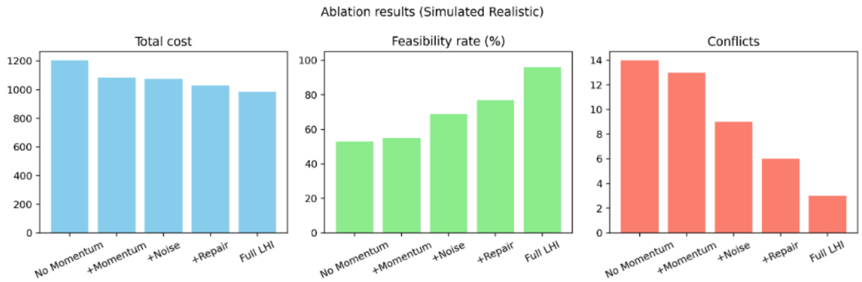

Figure 4. Comparison of ablation experiment results under different module combinations

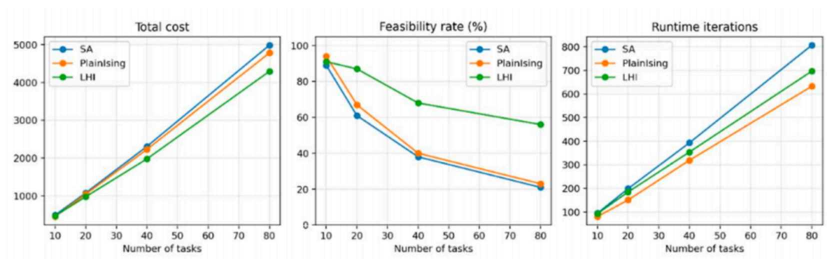

Figure 5. Performance trends of different methods under problem scale scaling conditions

4. Conclusion

This paper addresses the fundamental conflict in complex intelligent planning problems—namely, that discrete core decisions are amenable to Ising formulation, while complex execution constraints are difficult to encode uniformly within a single layer—and proposes a hierarchical hybrid Ising simulation framework. The proposed method maps task assignment and coarse-grained ordering decisions to an outer-layer QUBO/Ising energy minimization problem, while preserving path construction, conflict detection, temporal coordination, and local repair within an inner-layer module. Through a closed-loop mechanism of "outer-layer search–inner-layer repair–feedback update," cross-layer collaborative optimization is achieved. In the outer-layer solution process, an enhanced hybrid annealing mechanism is further constructed by incorporating continuous relaxation, momentum memory, noise perturbation, and feedback correction, thereby improving search stability, global exploration capability, and responsiveness to complex feasibility information.

Simulation results demonstrate that under varying map sizes, task scales, numbers of agents, and obstacle densities, the proposed method achieves superior or comparable performance relative to baseline methods in terms of total cost, feasible solution rate, conflict control, and convergence stability, with particularly pronounced advantages in highly complex, multi-constraint-coupled scenarios. These findings indicate that hierarchical modeling and closed-loop feedback mechanisms can effectively enhance the adaptability of Ising-based methods to complex intelligent planning problems, offering a feasible pathway toward practical applications such as multi-robot coordination, complex scheduling, and intelligent logistics.

Nevertheless, this study has certain limitations. For instance, the inner-layer repair remains primarily rule-based, the number of outer-layer ordering variables grows under large-scale task settings, and the feedback mechanism still has room for further adaptive optimization. Future research can be directed toward learningbased repair mechanisms, sparse modeling, adaptive annealing scheduling, and deployment on real optical/photonic Ising platforms [14, 15].

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