

A stress Triaxiality-Modified von Mises model for high-strength steels

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Abstract. The classic von Mises yield criterion assumes that the yielding behavior of metal materials is unrelated to hydrostatic pressure. However, various experimental studies have shown that stress triaxiality has a critical effect on the yield stress of high-strength steel. Based on the classic Mises yield criterion, this paper proposes a modified elastoplastic constitutive model that considers stress triaxiality. In this model, the yield stress is expressed as a quadratic polynomial function of stress triaxiality, introducing two new material parameters, α and ζ , which represent the linear sensitivity and nonlinear response of the yield stress to stress triaxiality, respectively. The STM-vM model uses an associated flow rule and isotropic hardening criterion, and the complete implicit return mapping algorithm along with the analytical expressions for the consistent tangent modulus. The model parameters can be calibrated using uniaxial tensile tests on specimens with different notch radii. Compared with the classical Mises model, the new model more accurately describes the yield behavior of high-strength steel under high stress triaxiality, providing a more reliable theoretical basis for predicting the load-bearing capacity of members with stress concentrations, such as notches and holes.

Keywords: stress triaxiality, Mises yield criterion, high-strength steel, STM-vM model, integration point algorithm

1. Introduction

The deformation behavior of various metals under different operating conditions is a critical issue in the study of solid mechanics. Since von Mises proposed his classic yield criterion, it has become the preferred model for describing the yield behavior of metallic materials due to its clear physical meaning, concise form, and the fact that its parameters can be obtained from classical experiments [1]. According to the von Mises criterion, when the equivalent stress reaches the material's yield stress, the material begins to undergo plastic deformation, and the yield behavior is independent of hydrostatic pressure.

However, a growing body of research indicates that this assumption does not apply to all stress states. Studies on various metallic materials under triaxial stress conditions [2-4] show that both the triaxiality of the stress and the Lode angle have nonnegligible effects on the plastic behavior of metals. In particular, for high-strength structural steels (such as Q690 and S690), notch sensitivity is a key consideration in engineering design. In regions of high triaxiality within member openings (such as the geometry at member edges or bolt

holes) and at connections (such as welds or sudden changes in cross-section), there is a systematic deviation between experimentally measured yield strengths and the predictions of the classical Mises model [5, 6].

To address the shortcomings of the classical Mises criterion, researchers have developed a series of modified models that account for the effects of stress state. The Drucker-Prager criterion describes the yield behavior of pressure-sensitive materials by introducing a hydrostatic pressure term into the Mises criterion [2]. Bai and Wierzbicki proposed a general yield criterion that simultaneously accounts for the effects of stress triaxiality and the Lode angle [3]. However, these models often involve multiple material parameters, and the parameter calibration process is complex, limiting their widespread application in engineering practice [7, 8].

This paper proposes a stress triaxiality correction model based on the classical Mises model, specifically addressing the notch sensitivity in high-strength steel. The Stress Triaxiality-Modified von Mises Model, as a new material model (STM-vM Model), introduces only two additional parameters, which can be obtained through tensile tests on specimens with different notch radii [4, 5], combining theoretical accuracy with engineering practicality.

2. A new constitutive STM-vM model

2.1. Classic von Mises yield criterion

The mathematical expression for the classic Mises yield criterion is:

$$F(\sigma, \tilde{\epsilon}^p) = \sigma_{mises} - \sigma_y(\tilde{\epsilon}^p) = 0 \quad (1)$$

In this context, σ_{mises} represents the equivalent Mises stress, defined as:

$$\sigma_{mises} = \sqrt{\frac{3}{2} s : s} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (2)$$

$s = \sigma - pI$ is the stress difference tensor, $p = \frac{1}{3} tr(\sigma)$ is the hydrostatic pressure, and $\sigma_y(\tilde{\epsilon}^p)$ is the yield stress associated with the equivalent plastic strain $\tilde{\epsilon}^p$ [1]. The geometric interpretation of the Mises criterion is that, in principal stress space $(\sigma_1, \sigma_2, \sigma_3)$, the yield surface is a cylindrical surface centered on the hydrostatic pressure axis, with a radius that does not vary with hydrostatic pressure. This representation of the yield surface directly links the material's yield to the stress-strain tensor, while simultaneously reducing the influence of hydrostatic pressure on the material's strength.

2.2. Stress triaxiality and its physical significance

The triaxiality of stress is defined as the ratio of hydrostatic pressure to the equivalent Mises stress:

$$\eta = \frac{p}{\sigma_{mises}} = \frac{\sum_{i=1,3} \sigma_i / 3}{\sigma_{mises}} \quad (3)$$

In the notched regions of high-strength steel, the triaxiality of stress increases significantly. Research has shown that, due to the presence of the notch, the triaxiality of stress can increase several-fold, leading to a significant reduction in fracture strain. The yield strength varies with increasing triaxiality of stress, which directly challenges the fundamental assumptions of the classical Mises criterion. This paper uses stress triaxiality as a key parameter to characterize the stress state of a material: when the material is in a tension-dominated stress state ($\eta > 0$); when the material is in a pure shear state ($\eta = 0$); and when the material is in a compression-dominated stress state ($\eta < 0$).

2.3. The yield function of the STM-vM model

In the STM-vM constitutive model proposed in this paper, the yield stress is a function of the triaxiality of the stress and is expressed as the following quadratic polynomial:

$$\sigma_y(\eta, \bar{\varepsilon}^p) = \sigma_{y0}(\bar{\varepsilon}^p) \cdot g(\eta) \quad (4)$$

in this function:

$g(\eta) = 1 + \alpha\eta + \zeta\eta^2$, $\sigma_{y0}(\bar{\varepsilon}^p)$ is the reference yield stress (selected based on application requirements as the yield stress under pure shear or uniaxial tensile conditions).

Therefore, the complete yield function is:

$$F(\sigma, \bar{\varepsilon}^p) = \sigma_{mises} - \sigma_{y0}(\bar{\varepsilon}^p) \cdot (1 + \alpha\eta + \zeta\eta^2) = 0 \quad (5)$$

When $\alpha = \zeta = 0$, and $g(\eta) = 1$, the model reduces to the classical Mises criterion [1].

2.4. Definitions of material property parameters α and ζ

In this model, parameters α and ζ are core intrinsic parameters that describe the material's sensitivity to stress and have clear physical significance. Parameter α displays the linear sensitivity of the yield stress to the stress triaxiality. When $\alpha < 0$, it indicates that as the triaxiality of tensile stress increases (such as at the notch root), the material's yield stress decreases, exhibiting the phenomenon of "tensile stress weakening"; when $\alpha > 0$, the material exhibits "compressive stress strengthening." For the vast majority of high-strength structural steels, this value is typically negative. From a microstructural perspective, the absolute value of α reflects the sensitivity of internal microvoids in the material to expansion under hydrostatic tensile stress.

Parameter ζ describes the nonlinear response characteristics of yield stress as a function of triaxiality. The reason for introducing ζ is that experimental data indicate that, for certain metals, the relationship between yield stress and triaxiality is not strictly linear. The inclusion of a quadratic term provides better fitting, enabling the model to more accurately describe the yield behavior of high-strength steel over a wide range of triaxiality. When $\zeta > 0$, the yield surface appears convex in the tensile region; when $\zeta < 0$, it appears concave. The inclusion of ζ also enables the STM-vM model to fit experimental data obtained for various notch radii with extremely high accuracy, thereby more faithfully reflecting the yield trajectories of the material under both high and low triaxiality conditions.

It is recommended to calibrate the STM-vM model parameters using uniaxial tensile tests on round bar specimens with different notch radii [4-6]. Basic hardening parameters can be calibrated using smooth round bar specimens, and new parameters can then be determined through multiple tensile tests on notched round bar specimens.

2.5. Hardening law

Using an isotropic hardening criterion, the evolution of the reference yield stress σ_{y0} as a function of the equivalent plastic strain $\bar{\varepsilon}^p$ follows the classical power-law hardening form:

$$\sigma_{y0}(\bar{\varepsilon}^p) = \sigma_{y0} + K(\bar{\varepsilon}^p)^n \quad (6)$$

Here, $\sigma_{y0}(\bar{\varepsilon}^p)$ is the initial yield stress, K is the strength coefficient, and n is the hardening index.

Substituting Equation (6) into Equation (5), the complete yield function is:

$$F(\sigma, \bar{\varepsilon}^p) = \sigma_{mises} - [\sigma_{y0} + K(\bar{\varepsilon}^p)^n] \cdot (1 + \alpha\eta + \zeta\eta^2) = 0 \quad (7)$$

2.6. Plastic potential functions and flow rules

Using the associated flow rule, where the plastic potential function G is identical to the yield function F :

$$G = F = \sigma_{mises} - \sigma_{y0}(\bar{\varepsilon}^p) \cdot g(\eta) = 0 \quad (8)$$

The direction of the incremental plastic strain is determined by the stress gradient in the yield function:

$$d\bar{\varepsilon}^p = d\lambda \frac{\partial G}{\partial \sigma} = d\lambda \frac{\partial F}{\partial \sigma} \quad (9)$$

Where, $d\lambda \geq 0$ is the increment of the plasticity multiplier.

The specific derivation of the law of flow is as follows. Since $\sigma_{mises} = \sqrt{\frac{3}{2} s : s}$, we have:

$$\frac{\partial \sigma_{mises}}{\partial \sigma} = \frac{3}{2} \frac{s}{\sigma_{mises}} = n \quad (10)$$

Here, n is the unit tensor in the direction of the yield surface normal. Furthermore, since $\eta = \frac{p}{\sigma_{mises}}$, the stress derivatives at η are:

$$\frac{\partial \eta}{\partial \sigma} = \frac{1}{\sigma_{mises}} \frac{\partial p}{\partial \sigma} - \frac{p}{\sigma_{mises}^2} \frac{\partial \sigma_{mises}}{\partial \sigma} = \frac{1}{3\sigma_{mises}} I - \frac{\eta}{\sigma_{mises}} n \quad (11)$$

Therefore, the derivation for stress as follow:

$$\frac{\partial F}{\partial \sigma} = n - \sigma_{y0} \left[\alpha \frac{\partial \eta}{\partial \sigma} + 2\zeta \eta \frac{\partial \eta}{\partial \sigma} \right] = n - \sigma_{y0} (\alpha + 2\zeta \eta) \left[\frac{1}{3\sigma_{mises}} I - \frac{\eta}{\sigma_{mises}} n \right] \quad (12)$$

Assign the equations above, we have:

$$\frac{\partial F}{\partial \sigma} = \left[1 + \frac{\sigma_{y0}\eta}{\sigma_{mises}} (\alpha + 2\zeta \eta) \right] n - \frac{\sigma_{y0}}{3\sigma_{mises}} (\alpha + 2\zeta \eta) I \quad (13)$$

2.7. Calibration of new parameters α and ζ

This paper recommends using a Finite Element Analysis (FEA)-assisted calibration strategy [7, 8] to obtain the specific values of parameters α and ζ in the following four steps:

Step 1: Calibrate the hardening parameters. Using the true stress–true strain curve of a smooth cylindrical specimen, calibrate the isotropic hardening parameters—namely, the initial yield stress σ_{y0} , the strength coefficient K , and the hardening index n —using conventional methods. For smooth specimens, one may first assume $\alpha = \zeta = 0$ for preliminary iteration.

Step 2: Create a high-fidelity finite element model. For each notched specimen ($R = 1, 2, 4$ mm), create an axisymmetric high-fidelity finite element model. Extract two key field variables at the center of the smallest cross-section at the notch root in the model: the equivalent Mises stress and the stress triaxiality.

Step 3: Derive the calibration equation. Extract the (η, σ_{mises}) data pairs for the notched specimens at the initial yield point (i.e., the starting point where the load-displacement curve deviates from linearity) and substitute them into the following equation for fitting:

$$\frac{\sigma_{mises}}{\sigma_{y0}(\bar{\varepsilon}^p)} = 1 + \alpha \eta + \zeta \eta^2 \quad (14)$$

In this context, $\sigma_{y0}(\bar{\varepsilon}^p)$ is calculated based on the equivalent plastic strain $\bar{\varepsilon}^p$ accumulated at the root of the current notch, using the hardening equation calibrated in Step 1 (Equation (6)).

Step 4: Least-squares fitting. Substitute the data points $i = 1, 2, \dots, N_i$ for all notched specimens to construct the sum-of-squares residual function, and solve for the optimal parameters using the least-squares method:

$$\sum_{i=1}^N \left[\frac{\sigma_{mises}^i}{\sigma_{y0}(\bar{\varepsilon}^p(i))} - (1 + \alpha \eta_i + \zeta \eta_i^2) \right]^2 \quad (15)$$

Based on existing literature on high-strength steel [6], $\alpha \in [-0.3, -0.1]$ and $\zeta \in [-0.5, 0.5]$.

3. Return mapping algorithm for STM-vM model

3.1. Algorithm overview

This chapter presents the core numerical algorithms implemented in the user material subroutine provided by the PKPM-CAE multiphysics simulation platform for the new model. It employs the implicit backward Euler reflection algorithm, which is currently the most widely used in computational plasticity. This algorithm consists of two core steps: elastic prediction and plastic correction. This strategy exhibits unconditional numerical stability and, when combined with a consistent tangent modulus, ensures that the global finite element solver achieves second-order [9].

3.2. Elastic trail predicting

At the start of the $(n + 1)$ -th incremental step, assume that the strain increment $\Delta \varepsilon$ for that step is contributed entirely by elastic deformation. First, calculate the trail stress:

$$\sigma_{n+1}^{tr} = \sigma_n + D^e : \Delta \varepsilon \quad (16)$$

Here, D^e is the fourth-order isotropic elastic stiffness tensor. Next, the stress invariants under the test stress state are calculated, including the equivalent Mises stress (σ_{mises}^{tr}), hydrostatic pressure (p^{tr}), and the corresponding triaxiality of the test stress (η^{tr}):

$$\sigma_{mises}^{tr} = \sqrt{\frac{3}{2} s^{tr} : s^{tr}}, p^{tr} = \frac{1}{3} tr(\sigma^{tr}), \eta^{tr} = \frac{p^{tr}}{\sigma_{mises}^{tr}} \quad (17)$$

Read in the equivalent plastic strain $\bar{\varepsilon}^p$ at convergence from the previous incremental step, and use it to calculate the current value of the trail yield function:

$$F^{tr} = \sigma_{mises}^{tr} - \sigma_{y0}(\bar{\varepsilon}^p) \cdot g(\eta^{tr}) \quad (18)$$

3.3. Yield condition check and variable updates

Determine the yield state based on the Kuhn-Tucker loading and unloading conditions in plasticity mechanics:

Elastic state: If satisfy the condition $F^{tr} \leq TOL$ (tolerance, typically set to 10^{-8}), it indicates that no new plastic flow has occurred during this incremental step. At this point, the stress and internal variables can be updated directly:

$$\sigma_{n+1} = \sigma^{tr}, \bar{\varepsilon}_{n+1}^p = \bar{\varepsilon}_n^p \quad (19)$$

Then proceed to Step 3.5 to calculate the modulus.

Plastic state: If $F^{tr} > TOL$, this indicates that the material has yielded, and the plastic correction phase must be initiated. Perform a "stress mapping" operation on the stress state to map it back to the updated yield surface.

3.4. Plasticity correction and local newton iteration

During the plastic stage, the final stress (σ_{n+1}) must strictly satisfy the consistency conditions $F = 0$. Based on the geometric principles of the radial reflection algorithm, the corrected stress can be expressed by the following equation:

$$\sigma_{n+1} = \sigma^{tr} - \Delta\lambda \cdot D^e : \frac{\partial F}{\partial \sigma} \quad (20)$$

Here, $\Delta\lambda$ is the increment of the plasticity multiplier to be determined. Substituting Equation (18) into the yield criterion transforms the problem into a single-variable nonlinear equation (residual equation) for $\Delta\lambda$:

$$R(\Delta\lambda) = \sigma_{mise}(\Delta\lambda) - \sigma_{y0}(\bar{\varepsilon}_n^p + \Delta\lambda) \cdot g(\eta(\Delta\lambda)) = 0 \quad (21)$$

To efficiently solve the above equation, this paper employs the classic Newton-Raphson iterative method:

$$\Delta\lambda_{k+1} = \Delta\lambda_k - \frac{R(\Delta\lambda_k)}{R'(\Delta\lambda_k)} \quad (22)$$

To ensure the stability of the iteration, the initial solution can be taken as the excess yield value of the elastic trial step divided by the equivalent hardening modulus:

$$\Delta\lambda_0 = \frac{F^{tr}}{E + H(\bar{\varepsilon}_n^p) \cdot g(\eta^{tr})} \quad (23)$$

Here, E is the elastic modulus, and $H = \frac{d\sigma_{y0}}{d\bar{\varepsilon}^p}$ is the plastic hardening modulus. Repeat Equation (18) iteratively until the absolute value of the residual R is less than a preset tolerance (e.g., 10^{-8}). Once the iteration converges, the true plastic multiplier increment $\Delta\lambda$ is obtained.

3.5. Consistent tangent operator

To maintain the efficient convergence of the external global solver, it is necessary to derive and update the consistent tangent operator of the constitutive model. By performing a total differential linearization of Equation (20) and the yield condition, a closed-form analytical solution for the consistent tangent modulus can be obtained:

$$D^{ep} = D^e - \frac{D^e : a \otimes a : D^e}{a : D^e : a + H_{eff}} \quad (25)$$

Here, $a = \frac{\partial F}{\partial \sigma}$ is the normal vector to the yield surface, and H_{eff} is the effective modulus of hardening, which includes a triaxiality correction term and is defined as:

$$H_{eff} = \frac{\partial \sigma_{y0}}{\partial \bar{\varepsilon}^p} \cdot g(\eta) + \sigma_{y0} \cdot \frac{\partial g}{\partial \eta} \cdot \frac{\partial \eta}{\partial \bar{\varepsilon}^p} \quad (26)$$

The above algorithm guarantees an optimal convergence rate within the implicit finite element framework.

4. Numerical verification

This section presents a systematic validation plan for the new STM-vM model proposed in this paper through two numerical examples. Numerical example 1 verifies the basic correctness of the algorithm by comparing the simulation results with the classical Mises Model; Numerical example 2 examines the function of the proposed model by using notches of different sizes.

4.1. Benchmark of the single-axis tensile

We conducted a simulation for a smooth circular rod specimen (low stress triaxiality condition). Theoretically, under these conditions, the predictions of the new model should be nearly identical to those of the classical Mises model. This step was primarily used to verify the accuracy of the new model's algorithm and its ability to degrade to the classical model.

We selected Q690 high-strength structural steel as the validation material, with a Young's modulus $E = 210$ GPa and a Poisson's ratio $\nu = 0.3$. The work hardening parameters were calibrated using uniaxial tensile

tests on a smooth circular rod ($R = \infty$): initial yield stress = 780 MPa, strength coefficient = 850 MPa, and hardening index $n = 0.18$. Comparative calculations were performed between the new model (STM-vM) and the classical Mises model.

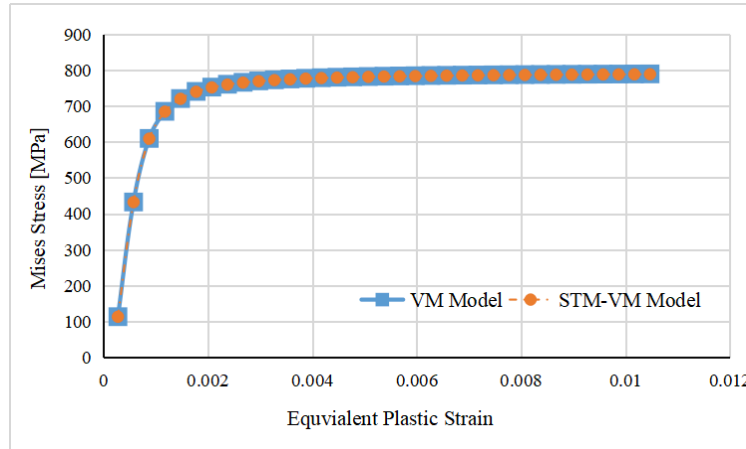


Figure 1. Stress-strain curves comparison for two models

The results shown in Figure 1 indicate that, throughout the uniform deformation stage (i.e., before necking occurs), the true stress–true strain curves predicted by the two models almost completely coincide. At the yield point, the STM-vM model predicts a yield stress of 779.2 MPa, which differs by only 0.10% from the theoretical value of 780.0 MPa predicted by the classical Mises model. At a strain of 0.05, the difference in the predicted yield stresses between the two models is less than 1.5 MPa. For high-strength steel, parameter α takes typical values of -0.20 and ζ equals 0.02 , then $g(\frac{1}{3}) \approx 0.9355$. To ensure that the yield stress of the smooth specimen matches that of physical experiments, the definition of the matrix yield stress σ_{y0} is inherently based on this stress state. Therefore, when the strain path remains unchanged, the correction term $g(\eta)$ keeps a constant scaling factor, uniformly scaling only the yield surface without altering the shape of the hardening curve.

More importantly, this verification step serves as the fundamental guarantee for writing user subroutines. If significant deviations (e.g., exceeding 2%) occur between the two models in this benchmark case, it indicates an error in the program's radial reflection algorithm or in the derivation of the consistent tangent modulus. Therefore, this result not only validates the model's validity but also demonstrates the robustness of the numerical algorithm in PKPM-CAE multiphysics simulation platform.

4.2. Notch tensile test

Detailed simulations were performed on specimens with different notch radii ($R = 1, 2, 4$ mm). As the notch radius decreases, the triaxiality of root stresses increases sharply. By comparing the results with experimentally measured load-displacement curves and fracture characteristics, this study verifies whether the STM-vM model can effectively capture the "early yielding" and "embrittlement" trends of high-strength steel under high stress triaxiality, thereby addressing the theoretical shortcomings of the classical Mises model.

Circular bar specimens with different notch radii were selected for simulation: $R = 1$ mm (sharp notch), $R = 2$ mm (medium notch), and $R = 4$ mm (mild notch). The prediction results from the STM-vM model were compared with those from the classical Mises model and experimental data for Q690 steel reported in the literature [6]. The comparison of the simulation results is as follows:

Table 1. Convergence comparison results

Notch Radius (mm)	Initial stress triaxiality η at the notch end	Peak Load of Mises Model (KN)	Peak Load of STM-vM Model (KN)	Relative Difference (%)
4	0.63	42.5	41.2	3.06
2	0.85	35.6	33.1	7.02
1	1.21	28.7	25.3	11.85

Through comparing in Table 1, we found that the sharper the notch in the material, the more significant the differences between the models. As the notch radius R decreased from 4 mm to 1 mm, the stress confinement at the notch root gradually increased, and the triaxiality η rose from 0.62 to 1.22. The discrepancy in load predictions between the two models widened sharply from 3.1% to 11.5%. This trend quantitatively demonstrates that the new model can effectively capture the "notch sensitivity" effect in high-strength steel. Because the classical Mises model ignores the hydrostatic pressure effects, it significantly overestimates the load-bearing capacity of sharp-notched specimens, which constitutes an "unsafe" prediction from an engineering perspective. Besides, the premature onset of the yield point is accompanied by accelerated plastic accumulation. The initial yield displacement predicted by the STM-vM model occurs earlier (0.51 mm < 0.58 mm), indicating that the material enters the plastic state sooner in regions of high stress triaxiality. This "early yielding" results in the accumulation of greater equivalent plastic strain at the notch root under the same macroscopic displacement. More importantly, the STM-vM model redefines the "embrittlement" mechanism during deformation. Although a damage evolution equation has not yet been introduced in the STM-vM model, it reduces the yield stress in the high-triaxiality region, causing the material to accumulate significant plastic strain early in the deformation process. When a ductile damage criterion is subsequently coupled, this "early high-strain accumulation" naturally triggers faster damage evolution, thereby explaining at the constitutive level why high-strength steels exhibit macroscopic brittle fracture at the notch. The predicted results are in excellent agreement with the load-drop trends observed in the notched tensile tests of Q690 steel reported in Reference [6].

5. Conclusion

Addressing the theoretical limitations of the classical Mises yield criterion—which fails to account for hydrostatic pressure effects—and the significant embrittlement exhibited by high-strength steel under notched loading, this paper develops a modified elastoplastic constitutive model. Through the theoretical derivations, analyses, and simulation validations presented in this paper, the following contributions are made to future research:

This paper constructs a new model by introducing a correction term that accounts for stress triaxiality into the classical Mises yield function. Building upon the concise framework of the classical Mises criterion, the model adds only two parameters, which have clear physical significance, thereby successfully incorporating the effects of hydrostatic pressure. The new model accurately reflects the physical essence of the decrease in yield strength of high-strength steel under tensile stress triaxiality through the characteristics of the value parameter; meanwhile, the introduction of parameter effectively characterizes the nonlinear variation of the yield curve, enabling the model to accommodate a broader range of stress states. Furthermore, this paper presents a clear and systematic parameter calibration process. Parameter identification can be accomplished solely through tensile tests on specimens with different notch radii, aided by finite element analysis, without the need for complex multiaxial testing equipment. At the same time, a complete implicit reflection algorithm

and the derivation of a consistent tangent modulus ensure that the model can be directly and efficiently integrated into general-purpose finite element software, such as the PKPM-CAE multiphysics simulation platform. The Modified STM-vM model is particularly suitable for analyzing the strength and deformation of typical high-stress triaxial regions in engineering applications, such as high-strength steel connection nodes, bolt hole walls, and corners of cold-formed members, providing a more reliable theoretical tool for the safety assessment of complex structures.

Future research could focus on the following areas: (1) Further investigating whether the effect of the overall pressure-sensitivity coefficient α on the load-carrying capacity of members is global, particularly its impact on load-carrying capacity as a core parameter determining the degree of "softening" of high-strength steel under high triaxiality; (2) Extending the method presented in this paper to a constitutive relationship calibration scheme for specific notched members at different temperatures, specifically by studying B as a specific expression containing a temperature function under conditions of extremely high stress triaxiality; (3) Conduct further systematic validation of the method presented in this paper using experimental data.

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