

To what extent can mathematical modelling be used to calculate the maximum cornering speed of a Formula 1 car?

Jiacheng Shi

Suzhou High School-SIP, Suzhou, China

3593113284@qq.com

Abstract. This study strongly focuses on the topic "To what extent can mathematical modelling be used to calculate the maximum cornering speed of a Formula One car", which focuses on the practical application of mathematical modelling in the analysis of F1 car dynamics. Firstly, through the literature review, this paper introduced related concepts in physics and engineering. Next, modelling with different complexities is sorted out. Factors determining maximum cornering speed in F1 cars are also briefly introduced. Secondly, models are established, from the basic model of a mass point, the rigid body and the Pacejka Magic Formula. Thirdly, this paper analyzed them by theoretical derivation and chose independent situations to calculate theoretical results. Finally, it compared the percentage error with practical data and discussed the applicability and limitations, respectively. The results show that, with the increase of the complexity of the model, percentage error gradually decreases, much closer to the actual situation. However, at the same time, it also needs higher requirements in data analysis and handling. Therefore, mathematical modelling has strong explanatory ability here, but its scope of application still depends on the rationality of model assumptions. The conclusion of this study has a certain reference value for understanding the dynamics of racing cars and also helps to apply physics and math to practical engineering problems, which has a positive significance for learning vehicle dynamics and engineering modelling.

Keywords: mathematical modelling, vehicle dynamics, F1 car, tyre mechanics

1. Introduction

The cornering speed of a Formula 1 car is more important than our common sense, because it strongly influences lap time and race results in a team. In contemporary competitive sport, modern Formula 1 cars are famous for higher racing speeds and advanced dynamic technologies. Thus, it has attracted enthusiastic fans. To win a Formula 1 championship, a racing team needs to consider different variances. Most people think that straight-line speed is visually impressive. Nevertheless, performances for cars in corners can determine the result of the game more intuitively and directly, which many people may neglect. Higher cornering speed reduces time lost in each corner and provides a potential advantage along the following ranking. As a result, using relevant methodologies to control or even improve cornering speed is crucial. Mathematical modelling, as a practical tool which plays a fundamental role in this process, allows engineers to estimate the situation without relying on physical testing which may cost much more time and money. Meanwhile, accurately

calculating maximum cornering speed remains challenging. For instance, the behavior of tyres, complex fluid motion and aerodynamics (e.g., aerodynamic forces vary with speed and vehicle attitude) and many parameters and variables are difficult to consider precisely [1].

Researches show that there are multiple models that have been developed to calculate maximum cornering speed. One of the simplest models is treating the car as a point mass using knowledge about circular motion and classical mechanics to carry out for a given corner radius [2]. Completely relying on Newton's classical mechanics (Newton's Laws) and models of different hardness in terms of mathematical and IT, it is possible to find insight into how factors such as speed, mass, tyre grip, and corner radius interact [2]. While these models are mathematically simple and useful for understanding fundamental relationships, they neglect important factors such as load transfer and downforce. To be improved, rigid body models were introduced. Milliken brothers wrote about load transfer, representing weight transfer [1]. It significantly improves the accuracy of cornering predictions. More advanced approaches involve multi-body models, where the car is modelled as interconnected rigid components, enabling further analysis of suspension movement and advanced aerodynamics [3]. In professional motorsport, computational models seem to be more precise and faster. Combining multi-body with database (e.g. obtained from Computational Fluid Dynamics (CFD) simulations or even graphical applications may have a fresher impression [4]. These methods provide high accuracy and are widely used in Formula 1. However, they require extensive computational resources and sophisticated technologies with advanced mathematical knowledge. As a result, such approaches are impractical for independent investigation at high school level for me. These challenges raise an important question, regarding how reliable and proper mathematical modelling can be used to calculate the maximum cornering speed of a Formula 1 car, particularly improving the accuracy and approaching the real value.

This research is going to focus on evaluating and finding out which kind of mathematical modelling can best calculate the cornering speed of a Formula 1 car, with much closer to a real value by comparing with 3 models (progressively harder to investigate: particle model, rigid body model and multi-rigid bodies model). Moreover, for each type, it will explore the assumptions required by each model and analysis different advantages and deficiencies. Through comparison with performance characteristics of Formula 1 cars, we can find the reliability and accuracy of different mathematical models, which after critically evaluated. This topic involves physics (mechanics, friction, centripetal force), mathematics (preliminary formula derivation, modelling, and functional relationships), and engineering (aerodynamics and racing design), which are closely related.

2. Research review

2.1. Fundamental physics of cornering

2.1.1. *Cornering speed*

Cornering speed is known as a maximal turning rate the object will have [2]. This document is a classic work in the field of vehicle dynamics, and the definition of cornering speed becomes the basic reference for subsequent car dynamics modelling. There are some factors that will affect speed, such as corner radius, road's superficial conditions, vehicle's initial velocity or even the tyres. In addition, Cornering also allows a sequence of slowing down, turning and accelerating. In common sense, if the vehicle ensures corner safely and efficiently, drivers should slow down before the turn, maintain a smooth line through the curve and accelerate out of the turn (this essay will mainly focus on constant speed because in F1 cornering, the change of acceleration is very small) [5]. As an authoritative document of vehicle handling dynamics, its analysis of cornering speed provides a key theoretical basis for the condition setting of F1 cornering speed modelling.

However, the consequence is totally different for F1 cars. In the actual cornering of F1 racing cars, the theoretical calculation of this cornering speed is still limited by many practical factors, making it tough for modelling as accurately as real cornering speed. To be illustrated, the aerodynamic characteristics of dynamic changes in racing cars, the nonlinear fluctuations of tire grip with tire temperature and road conditions. In addition, objective factors like the real-time effects of weather and driver operations, both add unstable variables to the modelling. This leads to that all mathematical models need to be constructed based on simplified assumptions, which naturally deviate from the real cornering dynamics process.

2.1.2. Circular motion

Circular motion is a key concept when considering cornering of racing cars. In Physics, circular motion is defined as the movement of an object along the circumference of a circle or rotation along a circular arc, the classic fundamental definition of circular motion in the field of physics. It can be uniform in a constant rate of rotation and constant tangential speed, or non-uniform (a changing rate of rotation). For example, orbital satellites around the Earth, a ceiling fan's blades rotating around a hub, or a ball that is tied to a rope and is being rotated in circles in the air [6]. As a general mechanics textbook, the examples in this literature effectively support the basic principles of circular motion, but in the analysis of the complex engineering object of F1 racing cars, more specific dynamic models need to be introduced. In a circular motion due to a constant change of the direction of the velocity, there must be acceleration by a centripetal force.

The Centripetal, in Latin *centrum*, means to seek [7], which is the force that makes a body follow a curved path [8]. This document serves as an authoritative reference for physics.

$$F_c = \frac{mv^2}{r} \quad (1)$$

Where F_c is the centripetal force, m is the mass, v is the linear velocity and r is radius.

For direction, it is always perpendicular to the motion of the body and towards the center point [9]. The definition of the direction of centripetal force in this literature is the fundamental geometric constraint for constructing the F1 cornering speed model. Sir Isaac Newton in the book, *the Principia: Mathematical Principles of Natural Philosophy*, proposed that "a force by which bodies are drawn or impelled, or in any way tend, towards a point as to a center" [10]. This literature, as the source of Newtonian mechanics, provides credibility. However, the centripetal force is named from the effect of applying due to the direction of the acceleration points to the center. Some properties are listed about the centripetal force:

1. Centripetal force is not a force with same type and certain properties. In contrast, force with any properties can be seen as a centripetal force, or even a component [9].

2. Centripetal force only changes the direction of the movement but remains the speed, even in a non-uniform circular motion [8].

In sum, Centripetal force can provide an important theoretical basis for analyzing the cornering speed of F1 cars. However, in real situations, a single centripetal force cannot accurately reflect the cornering speed because it is composed of multiple influencing factors, including complex aerodynamics, frictional forces, tire temperature, etc. Force majeure factors like weather changes, the car's vehicle control systems themselves also affect the simplified assumptions of mathematical models, and other factors. More importantly, human operational factors cannot be simply reflected in mathematical models, so although mathematical models based on circular motion can provide a theoretical maximum cornering speed, there are obvious limitations in predicting the turning speed of real racing cars.

2.1.3. Friction

Friction is the force which resists the relative motion between 2 solid objects' surfaces, fluid layers, and material elements sliding against each other [6, 8]. For example, if you are pushing the crate along the ground,

a force is impeding you from moving further. Generally, there are 4 classifications of friction force: static friction which prevents objects from static to move, kinetic friction which acts between a relatively moving object and a reference object, rolling friction between a rolling object and the surface it rolls on, and fluid friction which happens in liquids and gases. A general, almost fundamental formula, showing the relationship between friction and normal forces can be described as follows:

$$F_f \leq \mu F_n \quad (2)$$

Where F_f is the force of friction exerted by each surface on the other. It is parallel to the surface, in a direction opposite to the net applied force; μ is the coefficient of friction and an empirical property of the contacting materials; F_n is the normal force exerted by each surface on the other, directed perpendicular (normal) to the force. The Greek letter μ , which means the Coefficient of Friction (COF), is known as a dimensionless scalar value which equals the ratio of the force of friction between two objects and the force pressing them together, either during or at the onset of slipping. The coefficient of friction depends on the materials used, and it could be static friction coefficient or kinetic friction coefficient.

Therefore, in the actual cornering speed of racing cars, friction is actually one of the main sources of centripetal force. Based on the friction, a mathematical relationship can be established, thus carrying out the maximum cornering speed through friction. However, further problems in the actual calculation of friction emerge.

First, the friction coefficient is not a fixed constant in real states depends on tyres for most. Second, the accurate measurement of friction is affected by various practical factors including tire temperature, tire abrasion, contact materials with the ground, environmental changes and air pressure fluctuations. These complex and dynamic factors are too difficult to be fully incorporated into the general friction model, which leads to the fact that the maximum cornering speed calculated based on this model is much worse compared to the real cornering speed in actual F1 cars, which has also become a further discussion for analyzing the limitations of it in this study.

2.1.4. Drag coefficient

In a situation of fluid motion, the drag coefficient is a quantity that is used to quantify the drag or resistance of an object in a fluid environment. For example, air or water. It is used in the drag equation. A lower drag coefficient indicates that the object will have less aerodynamic or hydrodynamic drag. The drag coefficient is always associated with a particular surface area. The drag coefficient of any object comprises the effects of the two basic contributors to fluid dynamic drag: skin friction and form drag. The drag coefficient of a lifting airfoil or hydrofoil also includes the effects of lift-induced drag [11]. It is a classic essay in the field of aerodynamics, and its interpretation of the drag coefficient provides valuable reference for modelling in F1 cars. The drag coefficient of a complete structure such as an aircraft also includes the effects of interference drag.

$$C_d = \frac{2F_d}{\rho u^2 A} \quad (3)$$

Where F_d is the drag force, which is the force component in the direction of the flow velocity; ρ is the mass density of the fluid; u is the speed; A is the windward area. The reference area depends on what type of drag coefficient is being measured. For automobiles and many other objects, the reference area is the projected frontal area of the vehicle. This formula is the key foundation for quantifying F1 air resistance and modelling maximum cornering speed in this study.

Overall, the air resistance coefficient provides key aerodynamic parameters for constructing the mathematical model of the maximum cornering speed of F1. However, obvious limitations also exist.

Firstly, practical factors such as vehicle posture and workshop airflow interference make it difficult to accurately measure, increasing modelling errors. Secondly, it is worth to be mentioned that air resistance is not the core factor that restricts the bending speed of F1. The centripetal force provided by friction plays a decisive role. Therefore, this coefficient can only be an "assistant" in most mathematical modelling to improve the accuracy of calculating the maximum bending speed of F1. If much more advanced for the model, it still requires correction and other formulae.

2.1.5. Aerodynamics

Aerodynamics is the study of the motion of air, especially when the effect undergoes by a solid object [12]. A classic literature in the field of aerospace. It comprises themes in the sphere of fluid dynamics and its subfield of gas dynamics. The formal research of aerodynamics started in the eighteenth century. However, the fact is that some observations of fundamental concepts like aerodynamic drag were recorded much earlier.

By understanding the movement of air around an object (more academically called a flow field), scientists found and defined a lot of fundamental concepts. These concepts enable calculations of forces and moments acting on the objects. In most of the problems regarded as forces, there are several forces basically, including lift, drag, thrust and weight. It is worth to be mentioned that the first 2 forces are aerodynamic forces (lift and drag).

Aerodynamics are classified by the flow or properties of the flow, something like flow speed, compressibility, and viscosity. Aerodynamic problems can be categorized by flow speed relative to the speed of sound. Subsonic flows occur entirely below sonic speed, while supersonic is when flow exceeds sonic speed, and hypersonic flow refers to very high-speed regimes, generally above Mach 5, although its exact definition varies among aerodynamicists [13], an authoritative work on aerodynamics. For F1 to race as fast as possible, reducing drag and wind noise or even eliminating these issues is necessary and crucial. "For some classes of racing vehicles, it may also be important to produce downforce to improve traction and thus cornering abilities [14]."

From a theoretical perspective, a series of aerodynamic equations can be mathematically linked by combining with friction models, demonstrating the practical application value of mathematical modelling in analyzing the impact of aerodynamics on the maximum cornering speed of F1. However, changes in the car's posture, interference from the track environment airflow, and other complex flow fields, making it difficult to fully quantify and capture the aerodynamic characteristics. Therefore, although mathematical modelling can theoretically understand the influence of aerodynamics on the maximum cornering speed of F1, its ability ultimately depends on the model's consideration of the complexity of the real fluid environment.

2.2. Objective factors determining maximum speed in F1 cars

2.2.1. Tyre physics

Tyres play a fundamental role in determining maximum speed due to their grip, rolling resistance, and heat. The maximum cornering speed is majorly limited by the frictional force available between the tyre and the road surface (see 2.1.3 Friction). It is influenced by factors such as tyre compound, vertical load, temperature, and surface conditions [1]. Pirelli Motorsport, an Italian manufacturer which supplied F1 teams with tyres since 2011, has produced range of 18-inch tyres for the 2025 campaign that comprises six "slick" compounds [15]. Formula 1 tyres are designed with different rubber compounds, thus with different hardness. Each sort of tyre offers a trade-off between grip and durability. (from hardest to softest: the C1, C2, C3, C4, C5 and C6, with 'intermediates' and 'full wets' to match ordious weather conditions). It is worth mentioning that the C6 is a new ultra-soft compound and is expected to be used at certain street circuits. Drivers sampled it during a particular practice session at last year's Mexico City Grand Prix and the Abu Dhabi post-season test [16].

Soft tyres can provide superb performance interact with the road surface. This helps to improve the speed for F1 cars, but the main drawback is that they are easy to wear out. Compared to soft tires, hard tires are better durable, but this dramatically decreases the grip between the tyres and the road surface, which limits the cornering speed. Another core factor that influenced the performance is the temperature. It cannot be too low or too high for the temperature of the tyre, because both of them significantly decrease the grip. Moreover, the load for tires also makes an impact to the mechanical properties. If singly increasing the low transfer, it is unnecessary to bring a linear improvement for grip. The character constitutes a constraint condition by improving downforce and cornering performance. To be illustrated, doubling the load on a single tyre does not mean doubling the adhesion simultaneously. This phenomenon has been systematically demonstrated by Pacejka and discussed by Gillespie in academic studies [2, 17]. Vehicle mass is also a key factor that influences cornering performance. A lighter car can change direction more quickly and reduce the load on tyres during cornering. As Marmor notes, vehicle mass significantly affects handling, acceleration, and the overall dynamic behaviour of racing cars [18]. Although mass is not the main focus of this study, it remains an important background variable that affects the accuracy of mathematical models.

2.2.2. Engine

The engine provides the F1 car with the power to enable the car to run, and simultaneously to overcome the relevant resistance in the greatest extent. In today's Formula 1, the power unit is complicated because it combines both turbocharged engines and Energy Recovery Systems (ERS). This whole system not only allows the F1 car to have great power but also improves the efficiency of the usage of energy. The development of engines in F1 always attempts to find a better balance between these two advantages [19].

When the power provided by engines is large enough to overcome resistance from the air and other external forces, naturally the F1 car can go at its fastest speed on a track. The faster the speed of the car, the grater the increase in air drag upon it. As a result, even with a little increase in power from the engine, it also increases the speed of the car. However, due to the regulations of Formula 1, the usage of fuels and auxiliary power from ERS is strictly limited. This means that in all teams, the performance of the engines is almost same.

Overall, although the engine power is not the most significant factor to determine the speed of the F1 cars, it is still the "footstone" part because it allows the cars to move in the racing. When establishing the performance models for F1 cars, engines are always seen as a known condition usually, but not the key of speed. This is particularly evident in F1 racing.

2.2.3. Radius of corner

The radius of the corner is the major factor that affects the maximum cornering speed for F1 cars. For F1 cars to move in a curve, they do circular motion. As a result, the radius of the corner basically determines the greatest cornering speed. Some concepts of circular motion can be seen in 2.1.1.

From classical mechanics, the relationship between cornering speed v , corner radius r , and centripetal acceleration is given by $a = \frac{v^2}{r}$ [1]. This formula demonstrates that in the given centripetal acceleration, the maximum cornering speed will increase with the square root of the curve radius. Gillespie also points out that compared to a smaller curvature (reciprocal of radius), bigger curvatures put a stricter limit on maximum cornering speed [2]. Therefore, when the conditions of the grip and drag are similar, F1 cars can pass corners with bigger radii in a higher speed, whereas passing corners with smaller radii at a lower speed [1].

In summary, in real-world conditions, the effect of corner radius is closely linked to aerodynamics and tyre performance. In larger radii with higher speeds, aerodynamic downforce is stronger. This raises the normal on the tyres and allowing greater lateral force. By contrast, at small corner radii, lower speeds reduce aerodynamic effectiveness, making tyre grip the dominant limiting factor [4].

In conclusion, corner radius is a key parameter in mathematical models. Simple models often treat the radius as constant. While more advanced models consider successive changing radius (or curvatures). This improves the accuracy of the models because it may show instantaneous maximum speed during the curve.

2.3. Models

2.3.1. *Mathematical modelling*

Mathematical modelling has been widely applied in engineering physics and dynamics. They are used to comprehend and predict the regulations and laws in different kinds of motions. Among the researches about F1 cars, mathematical models can easily help researchers to simplify the complicated dynamic problems into frames that are easy to analyse [1]. They are particularly used to analyse the motion of cars, performances of engine and external factors like aerodynamics [4]. Discreetly promote the researches and upgrading of F1 car's performance and power.

For example, in the practical application, one of the mathematical models is used to estimate the single lap time in the Grand Prix. Other models may help to Optimize the energy of cars and make strategies for the power distribution. They even evaluate the influence brought by external forces and performance of driving. Research on vehicle dynamics has shown that simplifying models can predict the acceleration and cornering performance of racing cars. And for more complex models may Analyze effect of tyres and downforce deeply [20].

This paper is going to discuss models in three different types of complexity. Generally, the harder it is for the models, the smaller the error of theoretical calculation. This means that, the deeper the exploration for variables in different models, the closer the theoretical results obtained are to the real situation.

2.3.2. *Simple models*

Simple models rely on Newton's laws of motion [3], for instance, point mass. In this model, the car is seen as a point mass without physical dimensions, and this means that its shape, rotational inertia, suspension system, and detailed tyre behaviour are all ignored. Only the total mass of the vehicle and the external forces acting upon it are considered [1]. Obviously, it decreases the complexity of calculation in mathematics. It still provides basic analysis results, always seen as a start to a more complicated model. The point mass model is "a problem-solving technique in geometry which applies the physical principle of the center of mass to geometry problems involving triangles and intersecting cevians" [21].

There are some advantages. First, once you use basic mechanics knowledge, such as the balance of forces and Newton's laws, the maximum corner and speed can be calculated easily. Second, the required centripetal force is provided solely by the frictional force between the tyres and the road surface. Third, about the friction again, due to the Coulomb Friction Model, the friction coefficient is constant, which does not vary with temperature, load transfer or tyres [3].

Although this kind of model does not cost a lot of time and is easy to carry out, the theoretical calculation results are in lower precise. For example, it overlooks the aerodynamic force and other variables [1, 3].

Overall, it is highly meaningful since it clearly helps us to interpret some of the basic parameters. Furthermore, the particle model can also be seen as a reference to providing more details and corrections in advanced aerodynamic and computational models [4].

2.3.3. *Advanced models*

Next are more complicated advanced models. This kind of model will include drag, downforce and any forces which resist motion. Meanwhile, it introduces the influence caused by load transfer, to make sure that it is closer to a real physics state for F1 cars at high speed. Research indicates that adding drag significantly improves the accuracy of predictions, particularly for calculating the maximum speed and analyzing fuel

efficiency [22]. Some models also consider variable aerodynamic conditions, such as those caused by shape or turbulence when cars follow each other closely (always known as slipstream effects) [23]. In addition, in this type of model, downforce can be added by using a simplified aerodynamic equation:

$$L = \frac{1}{2} \rho C_L A v^2 \quad (4)$$

where L is downforce, ρ is air density, C_L is the lift coefficient (negative if for downforce, mentioned before in drag coefficient), and A is the reference area of the car.

Although for the most of the time the structures of these models are more sophisticated, they're still easy to use. Sometimes researchers also use multiple simple formulae, basic programming applications or other special applications. This aims to estimate the maximum cornering speed and be more realistic [20].

One of the intermediate models is the rigid body model. In this model, the Formula 1 car is no longer seen as a point mass, but as a rigid body with physical dimensions. This allows the model to be more realistic, because it adds the distribution of mass, rotational effects caused by load transfer, and vehicle stability based on the frame of classic mechanics [1].

In this approach, the car is assumed to have six dimensions, including longitudinal, lateral, vertical motion, and roll, pitch, and yaw rotations. During the cornering, the lateral acceleration produces a torque around the center of mass. This leads to the load transfers from inner tyres to outer tyres. The transfer will redistribute the maximum friction that each tire provides and influence the cornering speed. By adding these effects, rigid body models illustrate significant improvement compared to point mass models [3].

2.3.4. Further models

The relationship and interaction between tires and road surface is always a key point in vehicle dynamic research, especially when analyzing the ultimate cornering ability of racing cars. The accuracy of the lateral force of tires directly influences the research results. Traditional models used the Coulomb friction model ($F_{max} = \mu N$), which simply represents the friction times the friction coefficient. However, this model assumes a constant friction coefficient and is unable to depict complex changes of real tyres under the different downforce and turning angles [2]. With the development of vehicle dynamics research, Researchers start to use more accurate tyre models. Among these, one of the most widely used is an empirical model called the Pacejka Magic Formula.

The Pacejka model was initially proposed by Dutch scholar Hans B. Pacejka. It uses the empirical formulae series to fit the relationship between the force exerted on tyres and the slip angles carried out in experiments [17]. It compares relatively accurately in lateral forces form in different situations. By establishing parameters of B, C, D, and E to control the stiffness, shape, peak value, and curvature, thus enabling the model to describe the nonlinear variation of lateral force. Compared to simple models, the Pacejka model not only reflects the change in tyre grip under cornering but also the load sensitivity effect of tyres. It indicates that as the load on the tyre increases, the friction force provided per unit gradually decreases [1].

Due to its excellent fitting ability and efficiency in calculation, the friction Formula has become one of the most common tyre models in vehicle dynamics and is widely applied in fields such as racing engineering, vehicle stability control, and driving simulation systems [2]. In the study of racing dynamics, this model can more accurately predict the changes of forces exerted on tyres when in lateral accelerating. Thereby, it provides a more reliable theoretical value. However, this model is an empirical model, and its parameters usually need to be calibrated by using experimental data. So if in a situation without the testing values for real tyres, the accuracy of the model is still limited [17].

3. Discussion

After briefly introducing the relevant concepts of F1 car cornering speed and the influencing factors in the modelling process, this chapter will attempt to establish mathematical models to explore to what extent these models can match the real maximum cornering speed on the track, including formula establishment, calculation process, and error analysis.

In order to verify the extent of the modelling in the real F1 event scenario, this essay selects three typical curves of Bahrain track Turn 10 (low speed), Monza track Variante Ascari (medium speed) and Silverstone track Copse corner (high speed), and carries out quantitative calculation and error analysis combined with F1 official telemetry data and tire engineering parameters.

In this calculation (including subsequent models), C3 hard tires of the F1 2024 season are used as tire selection. The curvature of the corner used in the modelling of this study, including its accurate mapping value is the internal engineering confidential of FIA (2023) and F1 teams, has not revealed yet [24]. Thus, this paper uses the general equivalent reference radius in the realm [1], and the value is generally recognized in the industry, which is widely used in academic research. This equivalent value is only used for trend verification and analysis. Here are the data chosen below, after comparing

different sources and websites, as shown in Table 1.

Table 1. Trackrelated parameters and real measured maximum cornering speed for three F1 corners

Track Data of 3 Cornering			
Names of track	Radius R /m	Frictional coeff. μ	Real max cornering speed v / kmh^{-1}
Bahrain Turn 10	24	1.65	75
Monza Variante Ascari	75	1.60	162
Silverstone Copse	185	1.70	280

3.1. Simple model: point mass

In the simple model, this paper uses the point mass model to study the maximum cornering speed of the F1 car, ignoring the impact of complex factors on the cornering speed. In order to ensure the feasibility and simplicity of the analysis, the model is carried out under ideal conditions, and the following assumptions are made:

- I. The car is regarded as a particle.
- II. Centripetal force is completely provided by tire friction.
- III. The friction coefficient μ is a constant value.
- IV. Ignoring aerodynamic effects.

Next to the mathematical formulation. The centripetal force is provided by the lateral friction force from tyres:

$$F_c = \frac{mv^2}{R} \quad (5)$$

The lateral friction is given by:

$$F_{max} = \mu N \quad (6)$$

Positive normal force N is given by:

$$N = mg \quad (7)$$

Combining the 2 formulae and cancelling off mass m :

$$\mu mg = \frac{mv^2}{r} \quad (8)$$

$$v_{max} = \sqrt{\mu g R} \quad (9)$$

3.2. Analysis: point mass

Now, based on the data of 3 tracks and the formula deduced, here are the theoretical calculation processes corresponding to 3 different tracks and their percentage error. By the way, since the compared standard is real max cornering speed, the percentage error is calculated by: the theoretical results and percentage error for the point-mass model are presented as shown in Table 2.

$$\frac{|\text{theoretical value} - \text{real value}|}{\text{real value}} \times 100\% \quad (10)$$

Table 2. Theoretical calculation results and percentage error of the pointmass model

Theoretical Calculation for Simple Model		
Bahrain	$v_{max} = \sqrt{1.65 \times 9.81 \times 24} \approx 19.71 \text{ ms}^{-1}$ $\approx 70.96 \text{ kmh}^{-1}$	Percentage error: $\left \frac{70.96 - 75}{75} \right \times 100\% \approx 5.39\%.$
Monza	$v_{max} = \sqrt{1.60 \times 9.81 \times 75} \approx 34.31 \text{ ms}^{-1}$ $\approx 123.52 \text{ kmh}^{-1}$	percentage error: $\left \frac{123.52 - 162}{162} \right \times 100\% \approx 23.75\%.$
Silverstone	$v_{max} = \sqrt{1.70 \times 9.81 \times 185} \approx 55.56 \text{ ms}^{-1}$ $\approx 200.02 \text{ kmh}^{-1}$	percentage error: $\left \frac{200.02 - 280}{280} \right \times 100\% \approx 28.56\%.$

The percentage error of the three curves increases significantly with the increase in curve radius, which directly demonstrates the limitation. Firstly, the main reason for the rapid surge of percentage error is that the downforce under aerodynamics is completely ignored. In high-speed bends such as Silverstone Copse, the downforce greatly improves the vertical load and lateral grip of the tire, making the actual cornering speed much higher than the calculated value of the basic model [1]. Whereas in low-speed bending, the absolute value of downforce is small, so the error is relatively controllable.

Secondly, the nonlinear characteristics of the tire are oversimplified. Assumed friction coefficient μ is constant, but the friction coefficient of a real C3 hard tire will decrease slightly with the increase of vertical load, and the increase of tire temperature and the change of slip rate at high speed will further change the grip characteristics [15]. This nonlinear effect is more significant at medium and high cornering speed, which leads to the deviation between the predicted value and the real value.

In addition, the idealized assumptions of load transfer and curve geometry also introduce additional errors. When cornering, the vehicle's gravity center title leads to the redistribution of internal and external wheel loads, and the point mass assumption cannot reflect this dynamic process; Monza variant Ascari is a composite S-bend, and Silverstone corpse is a combination of transition curve and circular curve. The ideal circle radius assumption adopted by the model is approximate, which further increases the calculation deviation.

To sum up, the point mass model is only effective for the ideal scene of low speed, no downforce and uniform load distribution. For the medium and high-speed turns in F1 races, the prediction accuracy is seriously insufficient. It needs to be corrected by introducing downforce and using the Magic Formula to better fit the real dynamic conditions.

3.3. Advanced model: rigid body

Next is the part of "secondary models" by introducing new aerodynamic quantities and key concepts, downforce and load, much closer to a real-world physics of high-speed racing.

$$L = \frac{1}{2} \rho C_L A v^2 \quad (11)$$

Prerequisite assumptions:

- I. The car is seen as a rigid body (ignoring the rolling),
- II. Constant curvatures,
- III. The coefficient of friction between tyre and track μ is constant (linear);
- IV. Ignoring air drag force.

Second, the force which provides the centripetal force is the total horizontal friction, so the initial equation is:

$$\frac{mv^2}{R} = F_{total} \quad (12)$$

Third, according to the addition of downforce,

$$F_{total} = \mu N_{total} = \mu (mg + \frac{1}{2} \rho C_L A v^2) \quad (13)$$

Therefore,

$$\frac{mv^2}{R} = \mu mg + \mu \frac{1}{2} \rho C_L A v^2 \quad (14)$$

$$mv_{max}^2 = \mu R mg + \frac{1}{2} \mu R \rho C_L A v_{max}^2 \quad (15)$$

$$mv_{max}^2 + \frac{1}{2} \mu R \rho C_L A v_{max}^2 = \mu R mg \quad (16)$$

$$v_{max}^2 (m + \frac{1}{2} \mu R \rho C_L A) = \mu R mg \quad (17)$$

$$v_{max}^2 = \frac{\mu R mg}{(m + \frac{1}{2} \mu R \rho C_L A)} \quad (18)$$

$$v_{max} = \sqrt{\frac{\mu R mg}{m + \frac{1}{2} \mu R \rho C_L A}} \quad (19)$$

Here C_L is always negative!

Load transfer is defined as the distribution of forces from their point of application to other parts in a system, when the acceleration of the automobile occurs. When cornering, part of the weight of the inner wheels which are closer to the center of the circle, ΔW will transfer to the outer wheels, and the centrifugal force, which is the effect of the inertia, acts outwards. Its magnitude is equal to the centripetal force, $F = m \frac{v^2}{R} = ma$, where a is the lateral acceleration. Assume the forces are applied on the center of mass and the central of mass has height h . Thereby, a clockwise torque τ_1 will form. The pivot is at the contact part between outer wheels and the ground, seen as a point. h is the distance between F and pivot, therefore $\tau_1 = F \cdot h$. According to the conservation of moment, there must be an anticlockwise torque $\tau_2 = \tau_1$, apparently, load difference ΔW provides the force and the wheel track T provides the moment arm, $\tau_2 = \Delta W \cdot T$.

$$\tau_1 = \tau_2 \quad (20)$$

$$F \cdot h = \Delta W \cdot T \quad (21)$$

$$mah = \Delta W \cdot T \quad (22)$$

$$\Delta W = \frac{mah}{T} \quad (23)$$

This is the so-called "weight transfer equation".

For each side of the wheels, the weight is half of the total weight, $\frac{mg}{2}$. However, the normal force is $\frac{mg+L}{2}$ and L is the downforce mentioned before. For inner wheels, the load is transferred, so the normal force N_{inside} eventually is $\frac{mg+L}{2} - \Delta W$. For outer wheels, the load is added, so the normal force $N_{outside}$ is $\frac{mg+L}{2} + \Delta W$. Again, the centripetal force is equal to the total friction.

$$\frac{mR^2}{2} = \mu N_{total} = \mu(N_{inside} + N_{outside}) = \mu \left(\frac{mg}{2} - \Delta W + \frac{mg+L}{2} + \Delta W \right) = \mu(mg + L) \quad (24)$$

$$v_{max} = \sqrt{\frac{\mu R mg}{(m + \frac{1}{2} \mu R \rho C_L A)}} \quad (25)$$

3.4. Analysis: rigid body

Based on the rigid body of uniform circular motion, this model corrects the downforce and lateral friction. In order to ensure consistency with the point mass model, the previous data regarded to tracks and physical parameters are retained. In addition,

Mass of the entire car $m = 800$ kg,
Density of the air $\rho = 1.225$ kgm⁻³,
Windward area $A = 1.5$ m²,
Drag coefficient $C_l = -3.0$.

Again, all these data are from the industry and commonly used in academic research. The calculation outputs and percentage errors for the rigid-body model are summarised as shown in Table 3.

Table 3. Theoretical calculation results and percentage error of the rigidbody advanced model

Theoretical Calculation for the Advanced Model		
Bahrain	$v = \frac{1.65 \times 24 \times 800 \times 9.81}{800 + 0.5 \times 1.65 \times 24 \times 1.225 \times (-3.0) \times 1.5}^{\frac{1}{2}}$	Percentage error: $\left \frac{76.45 - 75}{75} \right \times 100\% \approx 1.81\%$
	$\approx 21.21 \text{ } ms^{-1}$	
	$\approx 76.35 \text{ } kmh^{-1}$	
Monza	$v = \frac{1.60 \times 75 \times 800 \times 9.81}{800 + 0.5 \times 1.60 \times 75 \times 1.225 \times (-3.0) \times 1.5}^{\frac{1}{2}}$	percentage error: $\left \frac{161.24 - 162}{162} \right \times 100\% \approx 0.47\%$
	$\approx 44.79 \text{ } ms^{-1}$	
	$\approx 161.24 \text{ } kmh^{-1}$	
Silverstone	In Silverstone, the denominator in the surd is negative ("-66.84"), indicates that under this high speed and radius, the linear assumption of the model has a theoretical boundary limitation, no more to use a mathematical modelling approach which is inconsistent with the physical law.	

In conclusion, the rigid body model has significant improvement in accuracy under low-speed (within 5% error) and medium speed conditions (within 1% error). However, under high-speed conditions, it indicates that the physical situation exceeds the limit of the theoretical mathematical model.

Firstly, the assumption of a constant drag coefficient is the major source of errors in high-speed operating conditions. The coefficient of real F1 cars will dynamically vary with vehicle speed, slip angle, and DRS status. In high-speed turns, drivers usually activate DRS to reduce resistance and downforce. The constant value assumption of the model cannot demonstrate this dynamic process, resulting in problems without real solutions under basic parameters [1].

Secondly, the assumption of a linear friction coefficient has not been fully corrected. The model still assumes a constant friction coefficient, so the loads transferred are cancelled out. This is the reason why it has no effect. At high speeds, the vertical load on the tires increases significantly, leading to a nonlinear reduction in the friction coefficient.

Above all, the modified rigid body model perfectly fits the derivation of F1 low-speed and medium speed tracks, with prediction accuracy far exceeding that of the point mass model. However, it is still limited by the core assumptions of linearity and steady state and cannot fully demonstrate nonlinear dynamic conditions of high-speed tracks. Further introduction of the Pacejka Magic Formula or dynamic aerodynamic models is needed to further improve prediction accuracy.

3.5. Further model: Pacejka Magic Formula

In order to more accurately describe the mechanical behaviour of the tyre during cornering, the Pacejka "Magic Formula" tyre model, which is widely used in vehicle dynamics research, can be introduced. This model was first proposed by Hans B. Pacejka, one of the empirical models widely used in modern vehicle dynamics. The Pacejka model describes the nonlinear relationship between tyre lateral force and slip angle through empirical formulas, and its basic form is usually written as:

$$F_y = D \cdot \sin \{C \cdot \arctan [Bx - E (Bx - \arctan(Bx))]\} \quad (26)$$

Where F_y represents the lateral force generated by a single tyre, x is the tyre slip angle parameter, and parameters B, C, D, and E stand for the stiffness factor, shape factor, peak factor, and curvature factor, respectively. Here are some data used from *The Dynamics modelling for Formula 1 Race Cars*, from Pacejka's team. "For Pirelli 18 inch F1 dry tire, use SAE j2452 standard test process; fitting results: C3 hard tyre B = 12.3, C = 1.82, D (Rated Peak Lateral Force) = 35200N, E = 0.86, F_0 (Rated Reference Vertical Load) = 20000N". These data are obtained based on publicly available literature, Pacejka's dissertation, and F1's overt tyre characteristics, then compiled and published into academic educational parameters designed for students modelling by Vehicle Dynamics Group from the university.

3.6. Analysis: Pacejka Magic Formula

However, this part of the calculations involves quite sophisticated theories in engineering and dynamics and exceeds initial expectations. It can be attributed to the following reasons. First, this model needs to recognize considerable empirical parameters and they must measure and fit in special experimental places, for example, tyre test bench with multiple, and cars' loads, slit angle [17]. Even if the group from Delft University has illustrated Different parameters for academic analysis, but these parameters need to calculate not simply by substitution but complex iteration and deduction. Second, the formula combined transcendental functions such as sine and arctangent, so the entire system cannot derive a "closed form" expression for the maximum cornering speed. In addition, the results of Magic Formula are divergent at extremely low speeds and lack the ability to describe transient dynamics and hysteresis phenomena [5]. This means that even the maximum cornering speed obtained through a numerical method, it still strongly depends on the reliability of input parameters and steady-state assumptions. Therefore, in the lack of complete tire experimental data and high fidelity of F1 simulation, this study cannot provide specific determined percentage error. The following discussion is based on the positive and negative viewpoints in existing literature and evaluates the erroneous range between the predicted value of the model and the actual cornering speed.

Supporters believe that the magic formula is one of the most reliable tyre models in the contemporary engineering industry. Pacejka in 2012 positioned it as an "effective tool" under steady-state conditions, who

definitely believes that it can fit experimental data well. This formula fits experimental data through complex mathematical functions and can accurately reproduce the mechanical properties of tyres. On the other hand, the formula itself is versatile because the key of the formula is a set of formulae with the same format can fully express the longitudinal force, lateral force, aligning torque, overturning force, drag torque, etc. of a tyre, and their variation patterns can be fitted by a group of trigonometric functions. Therefore, this method has been widely applied in engineering simulation. For example, in the field of racing engineering, Katz. J often uses this model to simulate and analyze the extreme handling performance of racing cars (*race car aerodynamics: designing for speed*).

Although Pacejka magic formula can accurately describe the nonlinear characteristics of tire lateral force, it still has some limitations and clear criticisms. Wong pointed out in "Theory of Ground Vehicles" that the predictive ability of pure empirical models is highly dependent on the experimental data range based on parameters [5]. In other words, when the vehicle operating conditions (such as vertical load or side slip angle) exceed this range, the accuracy of the model will significantly decrease. The technical report by SAE International also emphasizes that pure empirical models must be used with caution when analyzing vehicle dynamics under extreme conditions. Therefore, the maximum turning speed predicted by the model is still an idealized estimate, which needs to be further modified in combination with vehicle dynamics.

I take the view that I believe these 2 viewpoints are not contradictory. My interpretation is, the magic formula is indeed very accurate under general and ideal (steady state) conditions, which is why it is widely used. However, the limitations of the magic formula are obviously apparent, especially for extreme conditions such as high downforce, heavy load transfer, and tire performance in the high slip angle of F1 racing cars. The maximum cornering speed for F1 cars cannot be simply modeled in a mathematical way as same as the real one because the real situation is much more complicated beyond expectation. The Magic Formula is more like an 'advanced data fitting tool' rather than a theoretical model that can reveal the essence of physics. Therefore, in my research, I did not blindly rely on magic formulas to directly provide an accurate maximum cornering speed. If more accurate results are needed, they must be tested and calibrated with actual track data. That's why I chose to analyze these positive and negative viewpoints in the discussion section instead of giving a definite conclusion. Practice is the sole criterion for testing truth.

4. Conclusion

This study strongly focuses on the topic of "to what extent can mathematical modelling be used to calculate the maximum cornering speed of a Formula 1 car". The aim is to establish theoretical models at different levels to model, calculate, analyze, and evaluate the maximum cornering speed of racing cars, and to explore the differences between model prediction results and actual data. This question is not only seen as an academic exploration but also reflects the application of both physics and mathematics in practice.

In terms of research methods, this article first reviews the basic theories of mechanical concepts, model introductions, and influencing factors in related fields through my literature review. On this basis, relative models were constructed from simple to complex, which were particle models, advanced models considering rigid body characteristics and pressure under aerodynamics, and the Pacejka Magic Formula model. By comparing different models and combining industry recognized track data of curves of Bahrain T10 (low speed), Monza Ascari (medium speed), and Silverstone Copse (high speed), the theoretical calculation results were validated and error analyzed.

The research results indicate that a simple point mass model can predict the maximum cornering speed in low-speed curves well, with usually small errors and a certain reference value. However, in mid and high-

speed bends, the model significantly underestimates the actual speed, with errors ranging from 20% to 30%, and possibly even higher. This is mainly due to the model ignoring key factors such as aerodynamic pressure and tire nonlinear characteristics. Generally, as the complexity of the model increases, the results gradually reach a real situation. This illustrates that the method of mathematical modelling is still effective when analyzing the cornering performance of F1 cars.

From the perspective of practical applications, this research indicates that mathematical modelling can be an important tool to analyze the performance of F1 cars, especially very valuable in exploring engineer knowledge. By simplifying the problem, the theoretical maximum cornering speed of an F1 car can be quickly calculated, making it available and achievable. This research also demonstrates how to transfer abstract physical knowledge to detailed engineering problems. This is quite meaningful for comprehending vehicle dynamics and engineering model's establishment.

However, there still exist some limitations. Firstly, the track parameters used in the model, such as friction coefficient and curve radius, are mostly generally used in industry due to the lack of real data which are not revealed from FIA [25]. Precise experimental data cannot entirely support; Secondly, the models are based on ideal and uniform assumptions and do not consider the instantaneous behaviour of cars during the process of entering and exiting corners; In addition, for other details like suspension systems, mass distribution, multi-body models, and advanced aerodynamic analysis have not been deeply involved and discussed, which partially affects the error.

To address these issues, further investigation can be improved in several aspects below. First, technologies like telemetry and computational analysis tools (like MATLAB), can be introduced to fit experimental data. Experimental data can also be carried out to increase the reliability of results. Second, multi-body dynamics models or simulation methods can be better to analyse the instantaneous dynamics of vehicles. Third, by studying more specific knowledge in the university, exploring more models which can be easier to calculate the maximum cornering speed, much closer to real results. Overall, these improvements will further enhance the accuracy and reality of the results from models.

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